HERMITE

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7:51 AM

$$\left.\begin{array}{llll} x_0, & x_1, & \dots & , x_m \\ f(x_0), & f(x_1), & \dots, & f(x_m) \\ f'(x_0), & f'(x_1), & \dots & f'(x_m) \\ f''(x_0) & & \cdot & f''(x_m) \\ \vdots & & & \\ f^{(n_0)}(x_0) & & & \\ \hline n_0 & n_1 & & n_m \end{array}\right\} \quad \sum_{i=0}^{m} n_i - 1 = n$$

$P_n(x)$

$H_n(x)$

Dato: x_i, $i=0,\dots,m$, $x_i \neq x_j$, $i \neq j$

$P_n^{(k)}(x_i) = f_i^k$, $k=0,\dots,n_i-1$, $i=0,\dots,m$

$f_i^k = f^{(k)}(x_i)$

Lagranž: $n_i = 1$, $\forall i$

(T) Za proizvoljni skup realnih brojeva x_i i f_i^k, $k=0,\dots,n_i-1$, $i=0,\dots,m$ uz uslov da je $\forall 0 \le i,j \le m$, $x_i \neq x_j$ $\exists!$ polinom $P_n(x)$ stepena $n = \sum_{i=0}^{m} n_i - 1$ koji zadovoljava $f_i^k = P_n^{(k)}(x_i)$.

(D) Ako $\exists \Rightarrow$ jedinstven

pps. $\exists\ P_{n_1}(x) \neq P_{n_2}(x)$ HIP

$P_{n_1}^{(k)}(x_i) = f_i^k = P_{n_2}^{(k)}(x_i)$, $i=0,\dots,m$, $k=0,\dots,n_i-1$

$$Q(x) = \underbrace{P_{n_1}(x)}_{st\ n} - \underbrace{P_{n_2}(x)}_{st\ n} \Rightarrow st(Q) \le n$$

$$Q^{(k)}(x_i) = P_{n_1}^{(k)}(x_i) - P_{n_2}^{(k)}(x_i) = f_i^k - f_i^k = 0, \quad i=0,\dots,m$$

$\Rightarrow x_i$ bar n_i-tostruki koren polinoma $Q(x)$

$\Rightarrow$ broj nula $Q(x)$: $\sum_{i=0}^{m} n_i = n+1 > n$ ↯

egzistencija:

$P_n^{(k)}(x_i) = f_i^k \Rightarrow$ $n+1$ jednačina sa $n+1$ nepoznatom

$P_n(x) = \sum_{j=0}^{n} c_j x^j$, $c_j = ?$

Kramer: ✓ $\det(A) \neq 0 \Rightarrow \exists!$ rešenje

✗ $\begin{cases} \det(A) = 0 \wedge \det(\bar{A}) \neq 0 \Rightarrow \text{nema rešenja} \\ \det(A) = 0 \wedge \det(\bar{A}) = 0 \Rightarrow \infty \text{ mnogo rešenja ili nema rešenja} \end{cases}$

⊠

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$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$x_0 \quad x_0$

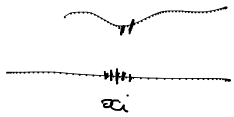
$f(x_0) \quad f'(x_0)$

$$f[x_0, x_0] = \frac{}{x_0 - x_0} \quad ↯$$

$\varepsilon > 0 \quad , \quad x_{ik}^{\varepsilon} \quad , k = 0, \dots, n_i - 1 \quad , i = 0, \dots, m$

razlicite

$\lim_{\varepsilon \to 0} x_{ik}^{\varepsilon} = x_i$



$x_{ik}^{\varepsilon} = x_i + k \cdot \varepsilon$

$f(x) = C^{n+1}[x_0, x_m] \longrightarrow P_n^{\varepsilon}(x)$

x	$f(x)$	$f[1]$	$\dots$	
x_{00}^{ε}	$f(x_{00}^{\varepsilon})$	$f[x_{00}^{\varepsilon}, x_{01}^{\varepsilon}]$		
x_{01}^{ε}	$f(x_{01}^{\varepsilon})$	$f[x_{01}^{\varepsilon}, x_{02}^{\varepsilon}]$		
$\vdots$	$\vdots$	$\vdots$		
$x_{0,n_0-1}^{\varepsilon}$	$f(x_{0,n_0-1}^{\varepsilon})$	$f[x_{0,n_0-1}^{\varepsilon}, x_{10}^{\varepsilon}]$		
x_{10}^{ε}	$f(x_{10}^{\varepsilon})$	$\vdots$		$f[x_{00}^{\varepsilon}, x_{01}^{\varepsilon}, \dots, x_{n,n_m-1}^{\varepsilon}]$
$\vdots$	$\vdots$	$\vdots$		
$x_{1,n_1-1}^{\varepsilon}$	$f(x_{1,n_1-1}^{\varepsilon})$			
$\vdots$	$\vdots$			
x_{m0}^{ε}	$f(x_{m0}^{\varepsilon})$			
$\vdots$	$\vdots$	$f[x_{m,n_m-2}^{\varepsilon}, x_{m,n_m-1}^{\varepsilon}]$		
$x_{m,n_m-1}^{\varepsilon}$	$f(x_{m,n_m-1}^{\varepsilon})$			

$$P_n^{\varepsilon}(x) = f(x_{00}^{\varepsilon}) + f[x_{00}^{\varepsilon}, x_{01}^{\varepsilon}](x - x_{00}^{\varepsilon}) + f[x_{00}^{\varepsilon}, x_{01}^{\varepsilon}, x_{02}^{\varepsilon}](x - x_{00}^{\varepsilon})(x - x_{01}^{\varepsilon})$$
$$+ \dots + f[x_{00}^{\varepsilon}, \dots, x_{0,n_0-1}^{\varepsilon}, x_{10}^{\varepsilon}](x - x_{00}^{\varepsilon}) \cdots (x - x_{0,n_0-1}^{\varepsilon})$$
$$+ \dots + f[x_{00}^{\varepsilon}, \dots, x_{n,n_m-1}^{\varepsilon}](x - x_{00}^{\varepsilon}) \cdots (x - x_{m,n_m-2}^{\varepsilon})$$

Veza PR i izvoda : $f[\underline{x}, x_0, \dots, x_n] = \dfrac{f^{(n+1)}(\xi)}{(n+1)!} \quad , \xi \in [\min(x_i), \max(x_i)]$

$$f[\underbrace{x_{ik}^{\varepsilon}, \dots, x_{il}^{\varepsilon}}_{l-k+1}] = \frac{f^{(l-k)}(\xi_{ikl}^{\varepsilon})}{(l-k)!} \quad , \xi_{ikl}^{\varepsilon} \in [\min(x_{ij}^{\varepsilon}), \max(x_{ij}^{\varepsilon})]$$

$\min(x_{ij}^{\varepsilon}) \xrightarrow{\varepsilon \to 0} x_i$, $\max(x_{ij}^{\varepsilon}) \to x_i$

$[x_i, x_i]$

$\varepsilon \to 0$: $\lim_{\varepsilon \to 0} x_{ik}^{\varepsilon} = x_i$

$\lim_{\varepsilon \to 0} \xi_{ikl}^{\varepsilon} = x_i$

$$\boxed{f[\underbrace{x_i, \dots, x_i}_{p+1}] = \frac{f^{(p)}(x_i)}{p!}} \quad , \quad p = l - k$$

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(T) Sve PR koje se javljaju u tablici imaju graničnu vrednost kad $\varepsilon \to 0$.

$q=0$: $f[o] = f(\cdot)$ ✓

$q-1 \overset{?}{\Rightarrow} q$: $f[x_{ik}^{\varepsilon}, \dots, x_{je}^{\varepsilon}] = \dfrac{f[x_{i,k+1}^{\varepsilon}, \dots, x_{je}^{\varepsilon}] - f[x_{ik}^{\varepsilon}, \dots, x_{j,e-1}^{\varepsilon}]}{x_{je}^{\varepsilon} - x_{ik}^{\varepsilon}}$

$\underline{i=j}$: $f[x_{ik}^{\varepsilon}, \dots, x_{ie}^{\varepsilon}]$ ✓ $f[\underbrace{x_i, \dots, x_i}_{p+1}] = \dfrac{f^{(p)}(x_i)}{p!}$

$\underline{i \neq j}$: imen. $\lim\limits_{\varepsilon \to 0} (\underset{\swarrow \atop x_j}{x_{je}^{\varepsilon}} - \underset{\downarrow \atop x_i}{x_{ik}^{\varepsilon}}) = x_j - x_i \neq 0 \quad (x_i \neq x_j)$

Brojilac: $f[x_{i,k+1}^{\varepsilon}, \dots, x_{je}^{\varepsilon}]$; $f[x_{ik}^{\varepsilon}, \dots, x_{j,e-1}^{\varepsilon}]$

PR reda $\underline{q}$

$\overset{IH}{\Rightarrow}$ ✓ ⊠

$P_n^{\varepsilon}(x) \xrightarrow{\varepsilon \to 0} P_n(x)$ HIP

$$P_n(x) = f(x_0) + f[x_0, x_0](x-x_0) + f[x_0, x_0, x_0](x-x_0)^2$$
$$+ \dots + f[\underbrace{x_0, \dots, x_0}_{n_0}](x-x_0)^{n_0-1} + f[\underbrace{x_0, \dots, x_0}_{n_0}, x_1](x-x_0)^{n_0}$$
$$+ f[x_0, \dots, x_0, x_1, x_1](x-x_0)^{n_0}(x-x_1) + \dots +$$
$$+ f[\underbrace{x_0, \dots, x_0}_{n_0}, \dots, \underbrace{x_m, \dots, x_m}_{n_m}](x-x_0)^{n_0} \cdot \ldots \cdot (x-x_m)^{n_m-1}$$

Ocena greške:

(I) Sa UNM: $f(x) - L_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} \cdot \omega_{n+1}(x)$

$x_{ij}^{\varepsilon} \in [y_1, y_2]$, $y_1 = \min(x, x_0, \dots, x_m)$

$y_2 = \max(x, x_0, \dots, x_m)$

$$f(x) - P_n^{\varepsilon}(x) = \frac{f^{(n+1)}(\xi^{\varepsilon})}{(n+1)!} \cdot \omega_{n+1}^{\varepsilon}(x), \quad \xi^{\varepsilon} \in [y_1, y_2]$$

$$\omega_{n+1}^{\varepsilon}(x) = \prod_{i=0}^{m} \prod_{j=0}^{n_i - 1} (x - x_{ij}^{\varepsilon})$$

$\varepsilon \to 0$. $\lim\limits_{\varepsilon \to 0} \omega_{n+1}^{\varepsilon}(x) = \lim\limits_{\varepsilon \to 0} \underbrace{(x - \underset{\downarrow x_0}{x_{00}^{\varepsilon}}) \cdot \ldots \cdot (x - \underset{\downarrow x_0}{x_{0,n_0-1}^{\varepsilon}})}_{(x-x_0)^{n_0}} \cdot \ldots \cdot \underbrace{(x - \underset{\downarrow x_m}{x_{m0}^{\varepsilon}}) \cdots (x - \underset{\downarrow x_m}{x_{m,n_m-1}^{\varepsilon}})}_{(x-x_m)^{n_m}}$

$$= \prod_{i=0}^{m} (x - x_i)^{n_i} = \omega_{n+1}(x)$$

$\lim\limits_{\varepsilon \to 0} f^{(n+1)}(\xi^{\varepsilon}) = z(x)$, $\quad f \in C^{n+1}[y_1, y_2]$

$$\min_{x \in [y_1, y_2]} f^{(n+1)}(x) \leq f^{(n+1)}(\xi^{\varepsilon}) \leq \max_{x \in [y_1, y_2]} f^{(n+1)}(x)$$

Teorema o sr. vr. $\Rightarrow \exists\, \xi \in [y_1, y_2]$ t.d. $\underset{z(x)}{f_{11}^{(n+1)}(\xi)}$

$$\boxed{f(x) - P_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} \cdot \omega_{n+1}(x)}$$

(II) Sa UNM: $f(x) - L_n(x) = f[x, x_0, \dots, x_n] \cdot \omega_{n+1}(x)$

$$f(x) - P_n^{\varepsilon}(x) = f[x, \underset{\downarrow x_0}{x_{00}^{\varepsilon}}, \dots, \underset{\downarrow x_0}{x_{0n_0-1}^{\varepsilon}}, \dots, \underset{\downarrow x_m}{x_{m0}^{\varepsilon}}, \dots, \underset{\downarrow x_m}{x_{m,n_m-1}^{\varepsilon}}] \cdot \omega_{n+1}^{\varepsilon}(x)$$

$\varepsilon \to 0$:

$$\boxed{f(x) - P_n(x) = f[x, \underbrace{x_0, \dots, x_0}_{n_0}, \dots, \underbrace{x_m, \dots, x_m}_{n_m}] \cdot \omega_{n+1}(x)}$$

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Veza PR i Izvoda: $f[x, x_0, \dots, x_n] = \frac{f^{(n+1)}(\xi)}{(n+1)!}$

$$f(x) - P_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} \cdot \omega_{n+1}(x) = f[x, \underbrace{x_0, \dots, x_0}_{n_0}, \dots, \underbrace{x_m, \dots, x_m}_{n_m}] \cdot \omega_{n+1}(x)$$

$$\Rightarrow f[x, x_0, \dots, x_0, \dots, x_m, \dots, x_m] = \frac{f^{(n+1)}(\xi)}{(n+1)!}$$

$x_0 \quad f(x_0) \quad f'(x_0) \quad f''(x_0) \dots$

$$P_n(x) = f(x_0) + f[x_0, x_0](x - x_0) + \dots + f[\underbrace{x_0, \dots, x_0}_{n_0}](x - x_0)^{n_0 - 1}$$

$$= f(x_0) + \frac{f'(x_0)}{1!} \cdot (x - x_0) + \dots + \frac{f^{(n_0-1)}(x_0)}{n_0!} \cdot (x - x_0)^{n_0 - 1}$$

Teilor

$+ \dots$

$f(x) - P_n(x)$

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(*) Napisati HIP za f-ju zadatu tablicom

	x	$f(x)$	$f'(x)$	$f''(x)$
x_0	-1	5	-3	-32
x_1	0	1	0	*
x_2	1	7	17	52

$n_0 = 3$
$n_1 = 2$
$n_2 = 3$

$n = \sum_{i=0}^{2} n_i - 1 = 3+2+3-1 = 7$

x	$f(x)$	$f[1]$	$f[2]$	$f[3]$	$f[4]$	$f[5]$	$f[6]$	$f[7]$
-1	5	-3	-16	$\frac{-1+16}{0+1} = 15$	-10	4	-1	1
-1	5	-3	-1	$\frac{4+1}{0+1} = 5$	-2	2	1	
-1	5	-4	$\frac{0+4}{0+1} = 4$	$\frac{6-4}{1+1} = 1$	2	4		
0	1	0	$\frac{6-0}{1-0} = 6$	$\frac{11-6}{1-0} = 5$	10			
0	1	6	$\frac{17-6}{1-0} = 11$	$\frac{26-11}{1-0} = 15$				
1	7	17	26					
1	7	17						
1	7							

$$f[x_0, x_0] = f[\underbrace{-1, -1}_{\substack{p+1 \\ p=1}}] = \frac{f'(-1)}{1!} = -3$$

$$f[x_0, x_1] = f[-1, 0] = \frac{f(0) - f(-1)}{0+1} = \frac{1-5}{+1} = -4$$

$$f[x_1, x_1] = f[0, 0] = \frac{f'(0)}{1!} = 0$$

$$f[x_1, x_2] = f[0, 1] = \frac{f(1) - f(0)}{1-0} = \frac{7-1}{1} = 6$$

$$f[x_2, x_2] = f[1, 1] = \frac{f'(1)}{1!} = 17$$

$$f[x_0, x_0, x_0] = f[\underbrace{-1, -1, -1}_{\substack{p+1=3 \\ p=2}}] = \frac{f''(-1)}{2!} = \frac{-32}{2} = -16$$

$$f[x_0, x_0, x_1] = \frac{f[x_0, x_1] - f[x_0, x_0]}{x_1 - x_0} = \frac{-4+3}{0+1} = -1$$

$$f[x_2, x_2, x_2] = f[1, 1, 1] = \frac{f''(1)}{2!} = \frac{52}{2} = 26$$

$$P_7(x) = 5 + (-3)(x+1) - 16(x+1)^2 + 15(x+1)^3 - 10(x+1)^3 \cdot x$$
$$+ 4(x+1)^3 \cdot x^2 - 1(x+1)^3 x^2 (x-1) + 1(x+1)^3 x^2 (x-1)^2$$

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nargin = Number ARguments IN

varargin = VARiable ARguments IN

function y= mojafa(varargin)

↓

cellarray

y= mojafun(1, [2,3], 45) (poziv iz komandnog. p.)

varargin{1} = 1

varargin{2} = [2,3]

varargin{3} = 45

kom. pro zor : .... ([-1, 5, -3, -32], [0,1,0], [1,7,17,52])

U .m : varargin{1} varargin{2} varargin{3}

yrep	f(1)	f(2)
~~8~~	~~-2~~	-16
~~5~~	-3	-1
~~8~~	-4	4
~~1~~	0	6
~~1~~	6	11
~~7~~	17	26
~~7~~	17	17
$\boxed{7}$	$\boxed{7}$	7

5 , -3

xrep	indeks
-1	1
-1	1
-1	1
0	2
0	2
1	3
1	3
1	3

Hornerova šema:

$2x^5 + 7x^4 + 3x^2 + 8x + 9$

$((((2x+7)x+0)x+3)x+8)x+9$

1. množi sa $(x-x_i)$
2. + PR(i)