Thursday, December 10, 2020
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LR metoda

$PA = PLU$, $L = \triangle$, $U = \triangledown$

$A = LU$

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1) $A_i = L_i \cdot U_i$, $L_i = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ * & & 1 \end{bmatrix}$, $U = \triangledown$

2) $A_{i+1} = U_i \cdot L_i$

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1) $A_i = L_i \cdot R_i$
2) $A_{i+1} = R_i \cdot L_i$

} LR metoda

(T) Ako $\exists$ sve dekompozicije $A_i = L_i \cdot R_i$ tada:

1) $A_{i+1} \sim A_i$, $A_{i+1} = L_i^{-1} A_i L_i$

2) $A_{i+1} = (L_1 \cdot \ldots \cdot L_i)^{-1} A_1 (L_1 \cdot \ldots \cdot L_i)$

3) $T_i = L_1 \cdot \ldots \cdot L_i$, $U_i = R_i \cdot \ldots \cdot R_1 \Rightarrow A^i = T_i \cdot U_i$

(D) 1) L_i^{-1} \ $A_i = L_i \cdot R_i$ / L_i

$L_i^{-1} A_i L_i = \underbrace{L_i^{-1} L_i}_{I} \underbrace{R_i L_i}_{A_{i+1}} \Rightarrow A_{i+1} \sim A_i$

2) $A_{i+1} \overset{(1)}{=} L_i^{-1} A_i L_i = L_i^{-1} (L_{i-1}^{-1} A_{i-1} L_{i-1}) L_i$

$= \ldots = L_i^{-1} L_{i-1}^{-1} \cdot \ldots \cdot L_1^{-1} \cdot A_1 L_1 \cdot \ldots L_i$

$= (L_1 \cdot \ldots \cdot L_i)^{-1} A_1 (L_1 \cdot \ldots \cdot L_i)$

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3) $A_{i+1} = (L_1 \cdots L_i)^{-1} A_1 (L_1 \cdots L_i)$

$(L_1 \cdots L_i) \cdot A_{i+1} = \underbrace{(L_1 \cdots L_i)(L_1 \cdots L_i)^{-1}}_{I} \cdot A_1 (L_1 \cdots L_i)$ (*)

T_i

$T_i \cdot U_i = L_1 \cdots \underbrace{L_i \cdot R_i}_{A_i} \cdots R_1 = \underbrace{L_1 \cdots L_{i-1} \cdot A_i}_{} \cdot R_{i-1} \cdots R_1$

$\overset{(*)}{=} A_1 \cdot (L_1 \cdots \underbrace{L_{i-1}) \cdot R_{i-1}} \cdots R_1$

$= A_1 \underbrace{L_1 \cdots L_{i-2} \cdot A_{i-1}} \cdot R_{i-2} \cdots R_1$

$\overset{(*)}{=} A_1^2 L_1 \cdots \underbrace{L_{i-2} \cdot R_{i-2}} \cdots R_1$

$= A_1^i \cdot \boxtimes$

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$$|\lambda_1| > \dots > |\lambda_n|$$

$e_1 = (1, 0, \dots, 0)^T$

$z_i = A^i \cdot e_1$

$Ax_k = \lambda_k x_k$

x_k cine baza

$e_1 = \alpha_1 x_1 + \dots + \alpha_n x_n$

$z_i = A^i(\alpha_1 x_1 + \dots + \alpha_n x_n) = \alpha_1 A^i x_1 + \dots + \alpha_n A^i x_n$ $\qquad A^i x_k = \lambda_k^i x_k$

$= \alpha_1 \lambda_1^i x_1 + \dots + \alpha_n \lambda_n^i x_n \qquad | : \lambda_1^i$

$\alpha_1 \neq 0$

$\lim_{i\to\infty} \frac{1}{\lambda_1^i} z_i = \alpha_1 x_1 + \alpha_2 \underbrace{\left(\frac{\lambda_2}{\lambda_1}\right)^i}_{\to 0} x_2 + \dots + \alpha_n \underbrace{\left(\frac{\lambda_n}{\lambda_1}\right)^i}_{\to 0} \cdot x_n \qquad / \lim_{i\to\infty}$

$|\lambda_1| > |\lambda_k|, k = 2, \dots, n$

$$\lim_{i\to\infty} \frac{1}{\lambda_1^i} z_i = \alpha_1 x_1$$

A) $z_i \sim \alpha_1 x_1$ (skoro kolineare nos+)

$A z_i \sim \alpha_1 A x_1$

$z_{i+1} \sim \alpha_1 \lambda_1 x_1$ $\qquad z_i = A^i \cdot e_1 \Rightarrow A \cdot z_i = \underbrace{A^{i+1} e_1}_{z_{i+1}}$

$z_{i+1} \sim \lambda_1 z_i$

$$z_i = A^i e_1 = T_i \cdot \underbrace{U_i \cdot e_1}$$

$$= T_i \cdot \begin{bmatrix} u_{11}^{(i)} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$= (t_1^{(i)} \dots t_n^{(i)}) \cdot \begin{bmatrix} u_{11}^{(i)} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$z_i = u_{11}^{(i)} \cdot t_1^{(i)}$$

$$A^i = T_i \cdot U_i$$

$$\begin{bmatrix} 1 & 0 \\ * & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & \square \end{bmatrix}$$

$$\begin{bmatrix} * & \dots & * \\ \vdots & \ddots & \vdots \\ * & 0 & * \end{bmatrix} * \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} u_{11} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$T_i = (t_1^{(i)} \dots t_n^{(i)})$$

$$U_i = R_i \cdot \dots \cdot R_1$$

$$\overset{R_i}{\square} \cdot \overset{R_{i-1}}{\square} \cdot \dots \cdot \overset{R_1}{\square} = \begin{bmatrix} * \\ 0 \end{bmatrix}$$

$$u_{11}^{(i)} = r_{11}^{(1)} \cdot \dots \cdot r_{11}^{(i)}$$

$$\Rightarrow \quad z_i = r_{11}^{(1)} \cdot \dots \cdot r_{11}^{(i)} \cdot t_1^{(i)}$$

$$t_1^{(i)} = (1, t_{21}^{(i)}, \dots, t_{n1}^{(i)})^T$$

$$z_i(1) = r_{11}^{(1)} \cdot \dots \cdot r_{11}^{(i)} \cdot 1$$

$x_1 = (x_{11}, \dots, x_{n1})^T$, $Ax_1 = \lambda_1 x_1$

$\boxed{x_{11} \neq 0}$

$$z_i(1) = r_{11}^{(1)} \cdot \dots \cdot r_{11}^{(i)}$$

$$z_{i+1}(1) = r_{11}^{(1)} \cdot \dots \cdot r_{11}^{(i)} \cdot r_{11}^{(i+1)}$$

$$\frac{z_{i+1}(1)}{z_i(1)} = r_{11}^{(i+1)} \qquad / \lim_{i \to \infty}$$

$$\lim_{i \to \infty} r_{11}^{(i+1)} = \lim_{i \to \infty} \frac{\lambda_1 \cdot z_i(1)}{z_i(1)} = \lambda_1$$

$$\boxed{\lambda_1 = \lim_{i \to \infty} r_{11}^{(i+1)}}$$

$z_i = r_{11}^{(1)} \cdots r_{11}^{(i)} \cdot t_1^{(i)}$, $\lim_{i\to\infty} \frac{1}{\lambda_1^i} z_i = \alpha_1 x_1$

$\underbrace{\lim_{i\to\infty} \frac{1}{\lambda_1^i} \cdot (r_{11}^{(1)} \cdots r_{11}^{(i)}) \cdot t_1^{(i)}}_{t_1 \neq 0} = \alpha_1 x_1 \neq 0$, $x_1 \neq 0$, $\alpha_1 \neq 0$

$At_1 = A(\alpha_1 x_1) = \alpha_1 A x_1 = \alpha_1 \lambda_1 x_1 = \lambda_1 t_1$

$\Rightarrow t_1$ je sopstv. vektor

$$\lim_{i\to\infty} R_i = \begin{pmatrix} \lambda_1 & & * \\ & \lambda_2 & \\ 0 & & \lambda_n \end{pmatrix}$$

$x_{11} = 0$:

$$z_i = \alpha_1 \lambda_1^i x_1 + \dots + \alpha_n \lambda_n^i x_n$$

$$z_i(1) = \alpha_1 \lambda_1^i \cdot \underbrace{x_{11}}_{0} + \alpha_2 \lambda_2^i x_{12} + \dots + \alpha_n \lambda_n^i x_{1n}$$

$$= 0 + \alpha_2 \lambda_2^i x_{12} + \dots + \alpha_n \lambda_n^i x_{1n} \qquad | : \lambda_1^i \qquad | \lim$$

$$\lim_{i\to\infty} \frac{1}{\lambda_1^i} z_i(1) = \lim_{i\to\infty} \alpha_2 \cdot \underbrace{\left(\frac{\lambda_2}{\lambda_1}\right)^i}_{\to 0} x_{12} + \dots + \alpha_n \underbrace{\left(\frac{\lambda_n}{\lambda_1}\right)^i}_{\to 0} \cdot x_{1n}$$

$$= 0$$

$$\frac{1}{\lambda_1^i} z_i(1) \xrightarrow{i\to\infty} 0$$

$$\Rightarrow \frac{1}{\lambda_1^i} z_i = \frac{1}{\lambda_1^i} \left(r_{11}^{(i)} \cdot \dots \cdot r_{1n}^{(i)}\right) t_1^{(i)}$$

$$\lim_{i\to\infty} \frac{1}{\lambda_1^i} z_i(1) = \lim_{i\to\infty} \frac{1}{\lambda_1^i} \cdot \left(r_{11}^{(i)} \cdot \dots \cdot r_{1n}^{(i)}\right) \cdot 1 = 0$$

$$\lim_{i\to\infty} \frac{1}{\lambda_1^i} z_i(k) = \underbrace{\lim_{i\to\infty} \frac{1}{\lambda_1^i} \left(r_{11}^{(i)} \dots r_{1n}^{(i)}\right)}_{\to 0} \cdot t_{k1}^{(i)} = \alpha_1 x_{k1}, \quad k = 2, \dots, n$$

$$\lim \frac{1}{\lambda_1^i} z_i = \alpha_1 x_1$$

$$\lim 0 \cdot t_k^{(i)} = \text{const}$$

$$x_1 = (0, \underbrace{x_{21}, \dots, x_{n1}}_{\exists \neq 0})^T$$

$$\Rightarrow t_{k1}^{(i)} \text{ divergiert}$$

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$\alpha_1 = 0 : \alpha_2 \neq 0$

$e_1 = \alpha_1 x_1 + \dots + \alpha_n x_n$

$\lim \frac{1}{\lambda_1^i} z_i = \alpha_1 x_1$ ovo nemamo

$\lim \frac{1}{\lambda_2^i} z_i = \alpha_2 x_2$ ovo imamo sad

$x_2 = (x_{12}, \dots, x_{n2})^T$

$x_{12} \neq 0$: ... ima (K)

$x_{12} = 0$: nema (K)

$\lim_{i \to \infty} r_{11}^{(i)} = \lambda_2$

t_2 sop. vektor

$X = (x_1 \dots x_n)$ kolone s. vekt.

$Ax_k = \lambda_k x_k$

$X^{-1} \mid AX = XD \quad / X^{-1} \quad D = \operatorname{diag}(\lambda_1 \dots \lambda_n)$

$X^{-1} A X X^{-1} = X^{-1} X D X^{-1}$

$X^{-1} A = D X^{-1}$, $Y = X^{-1}$

$YA = DY$, $y_k^T = (y_{k1}, \dots, y_{kn})$, $Y = (y_1^T \dots y_n^T)$ vrste

$y_j^T A = \lambda_j y_j^T$

(levi sop. vektor)

$y_j^T \cdot x_k = \begin{cases} 0, & j \neq k \\ 1, & j = k \end{cases}$ y_j^T ove baze

$y_j^T \mid e = \alpha_1 x_1 + \dots + \alpha_n x_n$

$y_j^T e = \alpha_1 y_j^T x_1 + \dots + \alpha_n y_j^T x_n = \alpha_j \cdot 1$

$y_{j1} = 0$
$\neq 0$