# Analiza 3

I smer

Vežbe

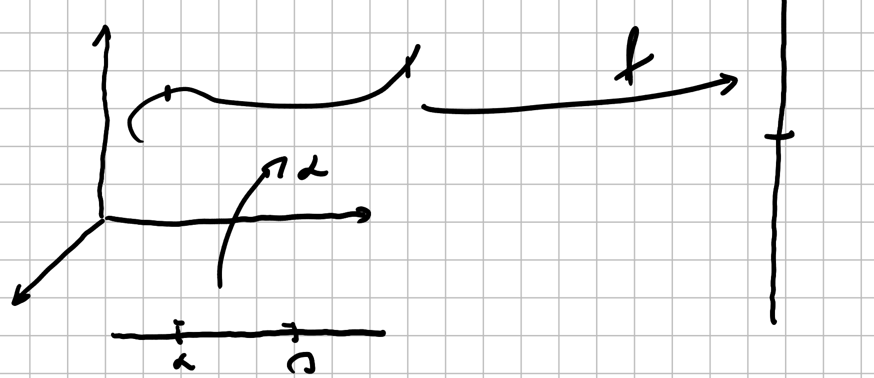
## Krivolinijski integrali

### Krivolinijski integral I vrste

Kriva $L = AB$

$$L(t) = (\varphi(t), \psi(t), \chi(t)) \qquad t \in [\alpha, \beta]$$

$$f : L \to \mathbb{R}$$




$f$ - definisana i ograničena na $L$

$\mathcal{P}: \alpha = t_0 < t_1 < t_2 < \dots < t_n = \beta$ podela $[\alpha, \beta]$

$\xi_i \in [t_{i-1}, t_i]$ - istaknuta tačka

$\xi = (\xi_1, \dots, \xi_n)$ - skup istaknutih tačaka

$$M_i = L(\xi_i)$$

$$M_i = (\varphi(\xi_i), \psi(\xi_i), \chi(\xi_i))$$

$\Delta s_i$ - dužina luka krive između $M_{i-1}$ i $M_i$

$\lambda(\mathcal{P}) = \max\limits_{1 \le i \le n} \{\Delta s_i\}$ - parametar podele

$$\sigma(L, f; \mathcal{P}, \xi) = \sum_{i=1}^{n} f(M_i)\, \Delta s_i$$

$I \in \mathbb{R}$ JE KRIVOLINIJSKI INTEGRAL I VRSTE AKO

$$\forall \varepsilon > 0 \quad \exists \delta > 0 \quad \forall P \quad \forall \xi \qquad \lambda(P) < \delta \Rightarrow |\sigma(L, f; P, \xi) - I| < \varepsilon$$

OZNAKE

$$\int_L f(x,y,z)\,dl \qquad \int_{AB} f(x,y,z)\,dl \qquad \int_A^B f(x,y,z)\,dl$$

(UMESTO $dl$ KORISTI SE I $ds$)

## OSOBINE:

1. $f, g$ DEFINISANE i $\int_L f\,dl$ i $\int_L g\,dl$ POSTOJE

1) LINEARNOST

$\alpha, \beta \in \mathbb{R}$
$$\int_L (\alpha f + \beta g)\,dl = \alpha \int_L f\,dl + \beta \int_L g\,dl$$

2) ADITIVNOST PO SKUPU

$C \in AB$ $\qquad$ $C$ IZMEĐU $A$ i $B$
$$\int_{AB} f\,dl = \int_{AC} f\,dl + \int_{CB} f\,dl$$

3) $$\left| \int_L f\,dl \right| \le \int_L |f|\,dl$$

4) $\forall M \in L \qquad f(M) \le g(M)$
$$\int_L f\,dl \le \int_L g\,dl$$

5) NE ZAVISI OD SMERA
$$\int_{AB} f\,dl = \int_{BA} f\,dl$$

6) TEOREMA O SREDNJOJ VREDNOSTI

$f \in C(L)$ $\qquad$ DUŽINA KRIVE $L$

$\exists M \in L \qquad \int_L f\,dl = f(M)\, s(L)$

RIMANOV INTEGRAL

7) RAČUNANJE
$$\int_{AB} f\,dl = \int_\alpha^\beta f(\varphi(t), \psi(t), \chi(t)) \sqrt{\varphi'(t)^2 + \psi'(t)^2 + \chi'(t)^2}\,dt$$

① $\int_L (x+y)\, dl$

L · GRANICA TROUGLA SA TEMENIMA

O (0,0) , A (1,0) i B (0,1)

$$I = \int_L (x+y)\, dl = \underbrace{\int_{OA} (x+y)\, dl}_{I_1} + \underbrace{\int_{AB} (x+y)\, dl}_{I_2} + \underbrace{\int_{OB} (x+y)\, dl}_{I_3}$$

$\underline{I_1}$ : $y=0$ $\quad dl = dx$ $\quad$ ( $x$ JE PARAMETAR $x'=1$ $\quad y'=0$ )

$$\int_{OA} (x+y)\, dl = \int_0^1 x\, dx = \frac{1}{2} x^2 \Big|_0^1 = \frac{1}{2}$$

$\underline{I_2}$ : $y = 1-x$ $\quad$ $x$ - PARAMETAR $\quad 0 \le x \le 1$

$y' = -1$

$$dl = \sqrt{1^2 + (-1)^2}\, dx = \sqrt{2}\, dx$$

$$\int_{AB} (x+y)\, dl = \int_0^1 (x+1-x)\sqrt{2}\, dx = \int_0^1 \sqrt{2}\, dx = \sqrt{2}$$

$\underline{I_3}$ : IZ SIMETRIJE $x$ i $y$ $\quad I_3 = I_1$

$$I = I_1 + I_2 + I_3 = \frac{1}{2} + \sqrt{2} + \frac{1}{2} = \boxed{1+\sqrt{2}}$$

② $\int_L y^2\, dl$

L : $x = a(t - \sin t)$ $\quad a>0$ $\quad t \in [0, 2\pi]$ - CIKLOIDA

$y = a(1 - \cos t)$

$x'(t) = a(1-\cos t)$ $\quad y'(t) = a \sin t$

$$dl = \sqrt{a^2(1-\cos t)^2 + a^2\sin^2 t}\; dt = a\sqrt{1-2\cos t+\cos^2 t+\sin^2 t}\; dt =$$

$$= a\sqrt{2}\sqrt{1-\cos t}\; dt = 2a\sqrt{\frac{1-\cos t}{2}}\; dt =$$

$$= 2a\left|\sin\frac{t}{2}\right| dt = 2a\sin\frac{t}{2}\, dt \qquad \frac{t}{2}\in[0,\pi]$$

$$I = \int_0^{2\pi} a^2(1-\cos t)^2\, 2a\sin\frac{t}{2}\, dt = 8a^3\int_0^{2\pi}\left(\frac{1-\cos t}{2}\right)^2\sin\frac{t}{2}\, dt$$

$$= 8a^3\int_0^{2\pi}\left(\sin\frac{t}{2}\right)^5 dt = \left[\begin{array}{l} u=\frac{t}{2} \\ du=\frac{1}{2}\,dt \end{array} \quad \begin{array}{c|c|c} t & 0 & 2\pi \\ \hline u & 0 & \pi \end{array}\right]$$

$$= 16a^3\int_0^{\pi}\sin^5 u\, du = 16a^3\int_0^{\pi}(1-\cos^2 u)^2\sin u\, du =$$

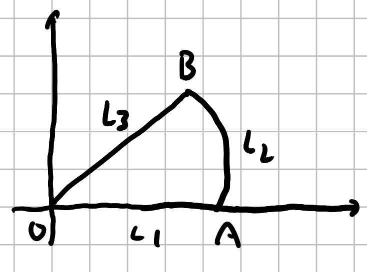
$$= \left[\begin{array}{l} z=\cos u \\ dz=-\sin u\, du \end{array} \quad \begin{array}{c|c|c} u & 0 & \pi \\ \hline z & 1 & -1 \end{array}\right] = 16a^3\int_{-1}^{1}(1-z^2)^2\, dz =$$

$$= 32a^3\int_0^1(1-2z^2+z^4)\, dz = 32a^3\left(z-\frac{2}{3}z^3+\frac{1}{5}z^5\right)\Big|_0^1$$

$$= 32a^3\left(1-\frac{2}{3}+\frac{1}{5}\right) = 32a^3\,\frac{8}{15}$$

③ $\displaystyle\int_L e^{\sqrt{x^2+y^2}}\, dl$

$L$ : RUB KRUŽNOG ISEČKA $0\le\rho\le a$, $0\le\varphi\le\frac{\pi}{4}$




$$L = L_1\cup L_2\cup L_3$$

$$\int_L e^{\sqrt{x^2+y^2}}\, dl = \underbrace{\int_{L_1} e^{\sqrt{x^2+y^2}}\, dl}_{I_1} + \underbrace{\int_{L_2} e^{\sqrt{x^2+y^2}}\, dl}_{I_2} + \underbrace{\int_{L_3} e^{\sqrt{x^2+y^2}}\, dl}_{I_3}$$

$\underline{I_1}$ : $y=0$ $x$-PARAMETAR $0\le x\le a$

$$dl = dx$$

$$\int_{OA} e^{\sqrt{x^2+y^2}}\, dl = \int_0^a e^x\, dx = e^x\Big|_0^a = e^a-1$$

$\underline{I_2}$: $\varphi$ - PARAMETAR

$x = a\cos\varphi \qquad y = a\sin\varphi \qquad 0 \le \varphi \le \frac{\pi}{4}$

$x' = -a\sin\varphi \qquad y' = a\cos\varphi$

$$dl = \sqrt{a^2\cos^2\varphi + a^2\sin^2\varphi}\, d\varphi = a\,d\varphi$$

$x^2 + y^2 = a$

$$I_2 = \int_{AB} e^{\sqrt{x^2+y^2}}\, dl = \int_0^{\pi/4} e^a a\, d\varphi = \frac{\pi}{4} e^a a$$

$\underline{I_3}$: $y = x \qquad x$ - PARAMETAR

$y' = 1 \qquad 0 \le x \le \frac{\sqrt{2}}{2} a$

$dl = \sqrt{2}\, dx$

$$I_3 = \int_{BO} e^{\sqrt{x^2+y^2}}\, dl = \int_0^{\frac{\sqrt{2}}{2}a} e^{\sqrt{2}x} \sqrt{2}\, dx = e^a - 1$$

$$I = I_1 + I_2 + I_3 = 2(e^a - 1) + \frac{\pi}{4} a e^a$$

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$$\int_L (x^2 + y^2 + z^2)\, dl$$

$L: \quad x^2 + y^2 + z^2 = a^2 \qquad x + y + z = 0$

$$\int_L (x^2 + y^2 + z^2)\, dl = \int_L a^2\, dl = a^2 \int_L dl = a^2 2\pi a = 2\pi a^3$$

OBIM VELIKOG KRUGA NA SFERI

5

$$\int_L \left(x^{\frac{4}{3}} + y^{\frac{4}{3}}\right) dl$$

$L: \quad x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ - ASTROIDA

$$\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{a}\right)^{\frac{2}{3}} = 1$$

$$\left(\left(\frac{x}{a}\right)^{\frac{1}{3}}\right)^2 + \left(\left(\frac{y}{a}\right)^{\frac{1}{3}}\right)^2 = 1$$

ZBIR DVA KVADRATA, ZGODNO JE DA JEDAN UZMEMO ZA $\cos t$, A DRUGI $\sin t$

$$\left(\frac{x}{a}\right)^{\frac{1}{3}} = \cos t \qquad \left(\frac{y}{a}\right)^{\frac{1}{3}} = \sin t \qquad t \in [0, 2\pi]$$

$$x = a\cos^3 t \qquad y = a\sin^3 t$$

$$x' = a\,3\cos^2 t(-\sin t) \qquad y' = a\,3\sin^2 t\cos t$$

$$dl = \sqrt{a^2 9\cos^4 t\sin^2 t + a^2 9\sin^4 t\cos^2 t}\,dt =$$

$$= 3a\sqrt{\sin^2 t\cos^2 t\,(\cos^2 t + \sin^2 t)}\,dt =$$

$$= 3a\,|\sin t\cos t|\,dt$$

$$I = a^{\frac{4}{3}}\int_0^{2\pi}(\cos^4 t + \sin^4 t)\,3a\,|\sin t\cos t|\,dt$$

$$= 12a^{\frac{7}{3}}\int_0^{\pi/2}(\cos^5 t\sin t + \sin^5 t\cos t)\,dt = \ldots$$

PODINTEGRALNA FUNKCIJA JE $\frac{\pi}{2}$ PERIODIČNA

⑥ IZRAČUNATI DUŽINU KARDIOIDE ZADATE SA

$$(x^2 + y^2 - 8x)^2 = 64(x^2 + y^2)$$

U POLARNIM KOORDINATAMA

$$(\rho^2 - 8\rho\cos\varphi)^2 = 64\rho^2$$

$$(\rho - 8\cos\varphi)^2 = 64$$

$$|\rho - 8\cos\varphi| = 8$$

1) $\rho \geq 8\cos\varphi$

$$\rho = 8 + 8\cos\varphi$$

$$\rho' = -8\sin\varphi$$

$\rho < 8\cos\varphi$

$$8\cos\varphi - \rho = 8$$

$$\rho = 8\cos\varphi - 8 < 0$$

$\lightning$

$$\rho \geq 0$$

$\sqrt{\rho}$ JE FUNKCIJA $\varphi$ $\qquad \rho = \rho(\varphi)$

$$x = \rho\cos\varphi \qquad y = \rho\sin\varphi$$

$$x' = \rho'\cos\varphi - \rho\sin\varphi \qquad y' = \rho'\sin\varphi + \rho\cos\varphi$$

$$dl = \sqrt{x'^2 + y'^2}\, d\varphi = \sqrt{(\rho'\cos\varphi - \rho\sin\varphi)^2 + (\rho'\sin\varphi + \rho\cos\varphi)^2}\, d\varphi$$
$$= \boxed{\sqrt{\rho^2 + \rho'^2}\, d\varphi}$$

S - DUŽINA KARDIOIDE L

$$S = \int_L dl = \int_0^{2\pi} \sqrt{(8+8\cos\varphi)^2 + (-8\sin\varphi)^2}\, d\varphi$$
$$= \int_0^{2\pi} \sqrt{64 + 128\cos\varphi + 64\cos^2\varphi + 64\sin^2\varphi}\, d\varphi$$
$$= \int_0^{2\pi} \sqrt{128 + 128\cos\varphi}\, d\varphi = \sqrt{256}\int_0^{2\pi}\sqrt{\frac{1+\cos\varphi}{2}}\, d\varphi$$
$$= 16\int_0^{2\pi} \left|\cos\frac{\varphi}{2}\right| d\varphi = \left| \begin{array}{c} t = \frac{\varphi}{2} \\ dt = \frac{1}{2}\, d\varphi \end{array} \quad \begin{array}{c|c|c} \varphi & 0 & 2\pi \\ \hline t & 0 & \pi \end{array} \right|$$
$$= 32\int_0^{\pi} |\cos t|\, dt = 32\left(\int_0^{\pi/2}\cos t\, dt - \int_{\pi/2}^{\pi}\cos t\, dt\right)$$
$$= 32\left(\sin t\Big|_0^{\pi/2} - \sin t\Big|_{\pi/2}^{\pi}\right) = \boxed{64}$$

## KRIVOLINIJSKI INTEGRAL II VRSTE

$J$ - INTERVAL

$\gamma : J \to \mathbb{R}^3$ PARAMETRIZOVANA KRIVA

$C = \gamma(J)$ NEPARAMETRIZOVANA KRIVA

$F : C \to \mathbb{R}^3$ VEKTORSKO POLJE

KRIVOLINIJSKI INTEGRAL II VRSTE JE

$$\int_C \vec{F}\cdot d\vec{r}$$

AKO JE ZATVORENA KRIVA $\oint_C \vec{F}\cdot d\vec{r}$

ORIJENTACIJA JE ZADATA RASTOM PARAMETRA

$$\int_C \vec{F}\, d\vec{r} = \int \vec{F}\,\frac{d\vec{r}}{dt}\, dt \leftarrow \text{RIMANOV INTEGRAL}$$

$$\vec{F} = (P, Q, R) \qquad d\vec{r} = (dx, dy, dz)$$

$$\int_C \vec{F}\cdot d\vec{r} = \int_C P\,dx + Q\,dy + R\,dz$$

$$\int_C P(x,y,z)\,dx = \int_\alpha^\beta P(x(t), y(t), z(t))\,\frac{dx}{dt}\,dt \qquad t\in I$$

OSOBINE:

- LINEARNOST
- ADITIVNOST
- ZAVISI OD ORIJENTACIJE

$$\int_{AB} \vec{F}\cdot d\vec{r} = -\int_{BA} \vec{F}\cdot d\vec{r}$$

1. $$\int_\gamma 2xy\,dx + x^2\,dy$$

$A = (0,0)$ $\quad B = (1,1)$

a) $\gamma$ - DUŽ $AB$

b) $\gamma$ - PARABOLA $y = x^2$

v) $\gamma$ - UNIJA DUŽI $AC$ i $CB$ $\quad C(1,0)$

a) $y = x \qquad 0\le x\le 1$

$dy = dx$

$$I = \int_\gamma 2xy\,dx + x^2\,dy = \int_0^1 2x^2\,dx + x^2\,dx = \int_0^1 3x^2\,dx = x^3\Big|_0^1 = 1$$

b) $y = x^2 \qquad 0\le x\le 1$

$dy = 2x\,dx$

$$I = \int_\gamma 2xy\,dx + x^2\,dy = \int_0^1 2x\cdot x^2\,dx + x^2\cdot 2x\,dx = \int_0^1 4x^3\,dx = x^4\Big|_0^1 = 1$$

v) $\gamma = \gamma_1 \cup \gamma_2$

$\gamma_1$ - DUŽ $AC$

$y = 0 \quad dy \qquad 0\le x\le 1$

$$\int_{\gamma_1} 2xy\,dx + x^2\,dy = \int_{\gamma_1} 0\,dx = 0$$

$\underline{\gamma_2}$ - DUŽ CB

$x = 1 \qquad 0 \le y \le 1$

$dx = 0$

$$\int_{\gamma_2} 2xy\,dx + x^2\,dy = \int_0^1 1\cdot dy = y\Big|_0^1 = 1$$

$$\int_{\gamma} 2xy\,dx + x^2\,dy = \int_{\gamma_1} 2xy\,dx + x^2\,dy + \int_{\gamma_2} 2xy\,dx + x^2\,dy = 1$$

( U SVA 3 SLUČAJA SMO DOBILI ISTU VREDNOST. VIDEĆEMO DA TO NIJE SLUČAJNO).

(2)

$$\oint_L (x+y)\,dx + (x-y)\,dy$$

$L: \quad (x-1)^2 + (y-1)^2 = 4$

$x = 1 + 2\cos\varphi \qquad y = 1 + 2\sin\varphi \qquad 0 \le \varphi \le 2\pi$

$dx = -2\sin\varphi\,d\varphi \qquad dy = 2\cos\varphi\,d\varphi$

$$\oint_L (x+y)\,dx + (x-y)\,dy =$$

$$= \int_0^{2\pi} (2 + 2\cos\varphi + 2\sin\varphi)(-2\sin\varphi)\,d\varphi +$$

$$+ (2\cos\varphi - 2\sin\varphi)\,2\cos\varphi\,d\varphi =$$

$$= -4\int_0^{2\pi} \sin\varphi\,d\varphi - 4\int_0^{2\pi} \cos\varphi\sin\varphi\,d\varphi - 4\int_0^{2\pi} \sin^2\varphi\,d\varphi +$$

$$+ 4\int_0^{2\pi} \cos^2\varphi\,d\varphi - 4\int_0^{2\pi} \cos\varphi\sin\varphi\,d\varphi =$$

$$= -4\int_0^{2\pi} \sin\varphi\,d\varphi + 4\int_0^{2\pi} (\cos^2\varphi - \sin^2\varphi)\,d\varphi - 4\int_0^{2\pi} 2\sin\varphi\cos\varphi\,d\varphi$$

$$= +4\,\cancel{\cos\varphi\Big|_0^{2\pi}}^{\,0} + 4\int_0^{2\pi} \cos 2\varphi\,d\varphi - 4\int_0^{2\pi} \sin 2\varphi\,d\varphi$$

$$= 2\sin 2\varphi\Big|_0^{2\pi} + 2\cos 2\varphi\Big|_0^{2\pi} = \boxed{0}$$

③ $I = \oint\limits_L y^2\,dx + z^2\,dy + x^2\,dz$

L : VIVIJANIJEVA KRIVA

$x^2+y^2+z^2=a^2 \qquad z \geq 0 \qquad x^2+y^2=ax \qquad a>0$

$$x^2 - ax + y^2 = 0$$
$$\left(x-\frac{a}{2}\right)^2 + y^2 = \left(\frac{a}{2}\right)^2$$

PARAMETRIZACIJA:

$x = \frac{a}{2} + \frac{a}{2}\cos t \qquad y = \frac{a}{2}\sin t$

$dx = -\frac{a}{2}\sin t\,dt \qquad dy = \frac{a}{2}\cos t\,dt$

$$z = \sqrt{a^2-x^2-y^2} = \sqrt{a^2 - \left(\frac{a}{2}+\frac{a}{2}\cos t\right)^2 - \left(\frac{a}{2}\sin t\right)^2}$$
$$= \sqrt{a^2 - \frac{a^2}{4} - \frac{a^2}{2}\cos t - \frac{a^2}{4}\cos^2 t - \frac{a^2}{4}\sin^2 t}$$
$$= \sqrt{\frac{3}{4}a^2 - \frac{a^2}{4}(\cos^2 t + \sin^2 t) - \frac{a^2}{2}\cos t} = \sqrt{\frac{a^2}{2} - \frac{a^2}{2}\cos t} =$$
$$= a\sqrt{\frac{1-\cos t}{2}} = a\left|\sin\frac{t}{2}\right| = a\sin\frac{t}{2}$$
$$dz = \frac{a}{2}\cos\frac{t}{2}\,dt$$

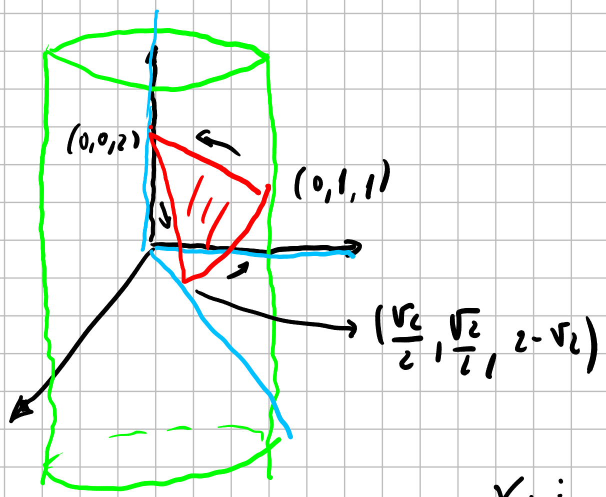
$$I = \int_0^{2\pi} \frac{a^2}{4}\sin^2 t\left(-\frac{a}{2}\sin t\right)dt + a^2\sin^2\frac{t}{2}\,\frac{a}{2}\cos t\,dt$$
$$+ \left(\frac{a}{2}+\frac{a}{2}\cos t\right)^2 \frac{a}{2}\cos\frac{t}{2}\,dt$$
$$= -\frac{a^3}{8}\int_0^{2\pi}\sin^3 t\,dt + \frac{a^3}{2}\int_0^{2\pi}\sin^2\frac{t}{2}\cos t\,dt$$
$$+ \frac{a^3}{8}\int_0^{2\pi}\cos\frac{t}{2}\,dt + \frac{a^3}{8}\int_0^{2\pi}\cos t\cos\frac{t}{2}\,dt + \frac{a^3}{8}\int_0^{2\pi}\cos^2 t\cos\frac{t}{2}\,dt$$
$$= \ldots$$

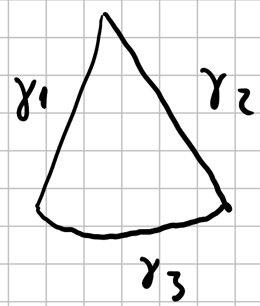
④ $\int\limits_\gamma \frac{xy}{x^2+y^2+1}\,dx + x\,dy + y\,dz$

$\gamma$ : PRESEK RAVNI $x+y+z=2$ SA RUBOM

TELA ZADATOG SA:

$x^2+y^2\le 1 \qquad 0\le x\le y$ (ORIJENTACIJA SA SLIKE)




$\gamma_1: \quad x=y$

$\gamma_2: \quad x=0 \qquad\qquad x+y+z=2$

$\gamma_3: \quad x^2+y^2=1$

$\gamma_1: \quad x=y=t \qquad z=2-y-z=2-2t \qquad 0\le t\le \frac{\sqrt2}{2}$

$dx=dy=dt \qquad dz=-2\,dt$

$$I_1=\int_{\gamma_1}\frac{t^2}{2t^2+1}\,dt+t\,dt+t(-2\,dt)$$

$$=\int_0^{\frac{\sqrt2}{2}}\frac{t^2\,dt}{2t^2+1}\,dt-\int_0^{\frac{\sqrt2}{2}}t\,dt=$$

$$=\frac12\int_0^{\frac{\sqrt2}{2}}\frac{2t^2+1-1}{2t^2+1}\,dt-\frac12 t^2\Big|_0^{\frac{\sqrt2}{2}}=\frac12\int_0^{\frac{\sqrt2}{2}}dt-\frac12\int_0^{\frac{\sqrt2}{2}}\frac{dt}{1+2t^2}-\frac12\,\frac{2}{4}$$

$$=\frac{\sqrt2}{4}-\frac14-\frac12\,\frac{1}{\sqrt2}\arctan\frac{t}{\sqrt2}\Big|_0^{\frac{\sqrt2}{2}}$$

$$=\frac{\sqrt2-1}{4}-\frac{1}{2\sqrt2}\arctan\frac12$$

$\gamma_2: \quad x^2+y^2=1 \qquad z=2-x-y$

$x=\cos t \qquad y=\sin t \qquad z=2-\cos t-\sin t \qquad \frac{\pi}{4}\le t\le\frac{\pi}{2}$

$dx=-\sin t\,dt \qquad dy=\cos t\,dt \qquad dz=(\sin t-\cos t)\,dt$

$$I_2=\int_{\pi/4}^{\pi/2}\frac{\cos t\sin t}{2}\,dt+\cos^2 t\,dt+\sin t(\sin t-\cos t)\,dt=$$

$$= \frac{1}{2}\int_{\pi/4}^{\pi/2} \cos t \sin t \, dt + \int_{\pi/4}^{\pi/2} (\underbrace{\cos^2 t + \sin^2 t}_{=1}) \, dt - \int_{\pi/4}^{\pi/2} \sin t \cos t \, dt$$

$$= \int_{\pi/4}^{\pi/2} dt - \frac{1}{2}\int_{\pi/4}^{\pi/2} \sin t \cos t \, dt$$

$$= t \Big|_{\pi/4}^{\pi/2} - \frac{1}{4} \sin^2 t \Big|_{\pi/4}^{\pi/2} = \left(\frac{\pi}{2} - \frac{\pi}{4}\right) - \frac{1}{4}\left(1^2 - \left(\frac{\sqrt{2}}{2}\right)^2\right) =$$

$$= \frac{\pi}{4} - \frac{1}{4}\left(1 - \frac{1}{2}\right) = \frac{\pi}{4} - \frac{1}{8}$$

$\gamma_3$: $\quad x = 0 \qquad y = t \qquad z = 2 - t \qquad 0 \le t \le 1$

$\qquad dx = 0 \qquad dy = dt \qquad dz = -dt$

$$\int_{\gamma_3} \frac{xy}{1+x^2+y^2} dx + x\,dy + y\,dz = \int_0^1 t(-dt) = -\frac{1}{2} t^2 \Big|_0^1 = -\frac{1}{2}$$

$$\int_{\gamma} \frac{xy}{1+x^2+y^2} dx + x\,dy + y\,dz = \int_{\gamma_1} \frac{xy}{1+x^2+y^2} dx + x\,dy + y\,dz +$$

$$+ \int_{\gamma_2} \frac{xy}{1+x^2+y^2} dx + x\,dy + y\,dz + \int_{\gamma_3} \frac{xy}{1+x^2+y^2} dx + x\,dy + y\,dz =$$

$$= \boxed{\frac{\sqrt{2}-1}{4} - \frac{1}{2\sqrt{2}} \arctan \frac{1}{2} + \frac{\pi}{4} - \frac{1}{8} - \frac{1}{2}}$$