ANALIZA 3

I SMER

VEŽBE

POVRŠINSKI INTEGRALI - NASTAVAK

TEOREMA: (STOKS)

γ - PROSTA, DEO PO DEO GLATKA ZATVORENA KRIVA

P - POVRŠ KOJOJ JE γ GRANICA

γ ORIJENTISANA TAKO DA JOJ JE P SA LEVE STRANE

$F = (P, Q, R)$ GLATKO POLJE NA $\overline{P}$.

TADA VAŽI:

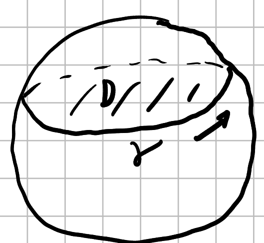
$$\oint_\gamma \vec{F} \cdot d\vec{r} = \oint_\gamma P\,dx + Q\,dy + R\,dz$$

$$= \iint_P \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) dy\,dz + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right) dz\,dx + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

OVA FORMULA POVEZUJE KRIVOLINIJSKI I POVRŠINSKI INTEGRAL I MOŽEMO PRELAZITI SA JEDNOG NA DRUGI I DA RAČUNAMO KOJI NAM JE LAKŠI. SPECIJALAN SLUČAJ JE GRINOVA FORMULA U RAVNI

1. $$\oint_\gamma y\,dx + z^2\,dy + x^2\,dz$$

γ : $x^2 + y^2 + z^2 = 4 \quad z = \sqrt{3}$ ORIJENTACIJA NA SLICI



γ JE KRUŽNICA NA SFERI.

PRIMETIMO DA JE γ GRANICA DISKA

$D: x^2+y^2 \le 1 \qquad z=\sqrt{3}$

MOŽEMO PRIMENITI STOKSOVU TEOREMU VODEĆI RAČUNA O ORIJENTACIJI

$P=y \qquad Q=z^2 \qquad R=x^2$

$\frac{\partial P}{\partial y}=1 \quad \frac{\partial P}{\partial z}=0 \quad \frac{\partial Q}{\partial x}=0 \quad \frac{\partial Q}{\partial z}=2z \quad \frac{\partial R}{\partial x}=2x \quad \frac{\partial R}{\partial y}=0$

$$\oint_\gamma y\,dx + z^2\,dy + x^2\,dz = \iint_D (0-2z)\,dy\,dz + (0-2x)\,dz\,dx + (0-1)\,dx\,dy$$

NAD D JE $z=\sqrt{3}$ KONSTANTNO PA JE $dz=0$

$$= \iint_D -\,dx\,dy = -\iint_D dx\,dy = -P(D) = -\pi$$

[KRIVA JE I GRANICA POVRŠI

$x^2+y^2+z^2=4 \qquad z \ge \sqrt{3}$

GORNJEG DELA LOPTE. MOGLO SE RADITI I PO OVOJ POVRŠI, ALI JE RAČUN KOMPLIKOVANIJI]

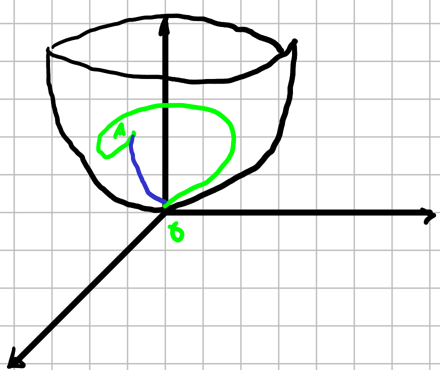
(2)

$$\int_{OA} yz\,dx + 3xz\,dy + 2xy\,dz$$

OA: $x = t\cos t$

$y = t\sin t \qquad t\in[0,2\pi]$

$z=t^2$



KRIVA PRIPADA PARABOLOIDU $z=x^2+y^2$, JER JE

$x^2+y^2 = (t\cos t)^2 + (t\sin t)^2 = t^2(\cos^2 t+\sin^2 t) = t^2 = z$

HOĆEMO DA PRIMENIMO STOKSOVU TEOREMU.

ALI KRIVA NIJE ZATVORENA.

$O = (0,0,0) \qquad A = (2\pi, 0, 4\pi^2)$

DA BISMO JE ZATVORILI MOŽEMO JE ZATVORITI PARABOLOM AO : $z = x^2, \ y = 0.$

NA AO JE $y = 0$ i $dy = 0$

$$\int_{AO} yz\,dx + 3xz\,dy + 2xy\,dz = 0$$

$$\int_{OA} yz\,dx + 3xz\,dy + 2xy\,dz =$$

$$\int_{OA} yz\,dx + 3xz\,dy + 2xy\,dz + \int_{AO} yz\,dx + 3xz\,dy + 2xy\,dz =$$

$$\iint_P (2x - 3x)\,dy\,dz + (y - 2y)\,dz\,dx + (3z - z)\,dx\,dy =$$

$$= \iint_P -x\,dy\,dz - y\,dz\,dx + 2z\,dx\,dy$$

GDE JE P DEO PARABOLOIDA OGRANIČEN KRIVAMA OA i AO.

NA P JE z FUNKCIJA x i y.

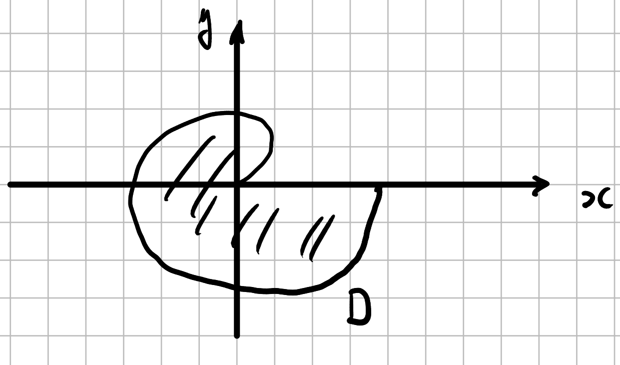
$$z = x^2 + y^2$$

$$\frac{\partial z}{\partial x} = 2x \qquad \frac{\partial z}{\partial y} = 2y$$

$$\iint_P -x\,dy\,dz - y\,dz\,dx + 2z\,dx\,dy =$$

$$= \iint_D (2x^2 + 2y^2 + 2(x^2 + y^2))\,dx\,dy = 4 \iint_D (x^2 + y^2)\,dx\,dy$$

GDE JE D PROJEKCIJA POVRŠI P NA RAVAN $z = 0$



GRANICA POVRŠI D JE KRIVA

$x = t\cos t \quad y = t\sin t \quad t \in [0, 2\pi]$

U POLARNIM KOORDINATAMA JE MOŽEMO ZAPISATI

$\rho = \varphi$

PA MOŽEMO INTEGRALITI SA GRANICAMA

$0 \le \rho \le \varphi \quad \varphi \in [0, 2\pi]$

$$I = 4 \iint_D (x^2+y^2)\, dx\, dy = 4 \int_0^{2\pi} \int_0^{\varphi} \rho^3\, d\rho\, d\varphi =$$

$$= 4 \int_0^{2\pi} \frac{1}{4} \rho^4 \Big|_0^{\varphi} d\varphi = \int_0^{2\pi} \varphi^4\, d\varphi =$$

$$= \frac{1}{5} \varphi^5 \Big|_0^{2\pi} = \boxed{\frac{32}{5} \pi^5}$$

NAREDNA TEOREMA POVEZUJE POVRŠINSKI I TROSTRUKI INTEGRAL:

TEOREMA: (GAUS)

$T \subseteq \mathbb{R}^3$ - KOMPAKTAN SKUP

S - GRANICA T, REGULARNA POVRŠ

$F = (P, Q, R)$ - GLATKO VEKTORSKO POLJE NA T

TADA VAŽI:

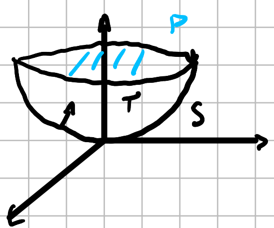
$$\iint_S P\, dy\, dz + Q\, dz\, dx + R\, dy\, dz = \iiint_T \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\right) dx\, dy\, dz$$

ORIJENTACIJA POVRŠI JE DA JE VEKTOR NORMALE USMEREN VAN TELA T.

③ $\iint_{S^+} y\, dz\, dx$

S^+: $\quad z = x^2+y^2 \quad 0 \le z \le 2 \quad$ GORNJA STRANA

(OVAJ ZADATAK JE VEĆ URAĐEN PARAMETRIZACIJOM)



DA BI PRIMENILI GAUSOVU FORMULU MORAMO DODATI „POKLOPAC" P TDJ. $S \cup P$ GRANICA TELA T.

TAKOĐE DA BI USKLADILI ORIJENTACIJE RAČUNAMO INTEGRAL PO S^-.

$$\iint_{S^-} y\,dz\,dx + \iint_P y\,dz\,dx = \iiint_T dx\,dy\,dz$$

NA P JE $z=2$ KONSTANTA PA JE $dz=0$;

$$\iint_P y\,dz\,dx = 0$$

$$\iint_{S^+} y\,dz\,dx = -\iint_{S^-} y\,dz\,dx = -\iint_{S^-} y\,dz\,dx - \iint_P y\,dz\,dx =$$

$$= -\iiint_T dx\,dy\,dz = -\iint_D \int_{x^2+y^2}^{2} dz\,dx\,dy =$$

$$= -\iint_D (2-x^2-y^2)\,dx\,dy =$$

D PROJEKCIJA NA RAVAN $z=0$ $\quad x^2+y^2 \leq 2$

$$= -\iint_{\substack{0\leq\rho\leq\sqrt{2}\\0\leq\varphi\leq 2\pi}} (2-\rho^2)\rho\,d\rho\,d\varphi =$$

$$= -\int_0^{2\pi} d\varphi \int_0^{\sqrt{2}} (2\rho-\rho^3)\,d\rho = -\varphi\Big|_0^{2\pi} \left(\rho^2 - \frac{1}{4}\rho^4\right)\Big|_0^{\sqrt{2}} =$$

$$= -2\pi\left(2-\frac{1}{4}4\right) = \boxed{-2\pi}$$

(4) $\displaystyle\iint_S \vec{F}\,d\vec{S}$

$$\vec{F} = (x-3y+2z,\ -3x+2y+z,\ 2x+y-3z)$$

$S:\quad x^2+y^2+z^2=4,\ z\geq\beta,\ \beta<0$, SPOLJNA STRANA

OVDE MOŽEMO RAZLIKOVATI 2 SLUČAJA

$1^\circ \quad \beta \leq -2$

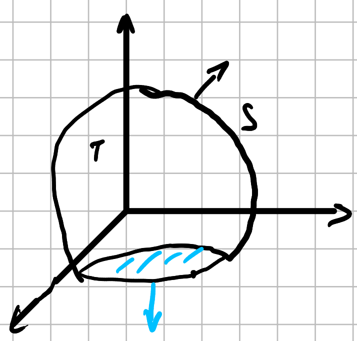
S JE U OVOM SLUČAJU RUB SFERE, I MOŽEMO PRIMENITI GAUSOVU FORMULU

$$\iint_S \vec{F}\cdot d\vec{S} = \iint_S (x-3y+2z)\,dy\,dz + (-3x+2y+z)\,dz\,dx + (2x+y-3z)\,dx\,dy =$$

$$= \iiint_T (1+2-3)\,dx\,dy\,dz = \iiint_T 0\,dx\,dy\,dz = 0$$

$2^\circ \qquad 0 > B > -2$

S U OVOM SLUČAJU NIJE GRANICA TELA, VEĆ MORAMO DOPUNITI POKLOPCEM P TJ. $S \cup P$ GRANICA.



$$\iint_S \vec{F}\cdot d\vec{S} + \iint_P \vec{F}\,d\vec{S} = \iiint_T 0\,dx\,dy\,dz = 0$$

$$\iint_S \vec{F}\,d\vec{S} = -\iint_P \vec{F}\,d\vec{S}$$

NA P JE $z = B \qquad x^2+y^2 = 4 - B^2 \qquad dz = 0$

$$\iint_P \vec{F}\,d\vec{S} = \iint_P (2x+y-3B)\,dx\,dy$$

$$= -\iint_D (2x+y-3B)\,dx\,dy$$

D PROJEKCIJA P NA RAVAN $z=0$

ZNAK $-$ JE ZBOG ORIJENTACIJE.

$$\iint_S \vec{F}\,d\vec{S} = \iint_D (2x+y-3B)\,dx\,dy$$

$$= 2\underbrace{\cancel{\iint_D x\,dx\,dy}}_{0} + \underbrace{\cancel{\iint_D y\,dx\,dy}}_{0} - 3B\iint_D dx\,dy$$

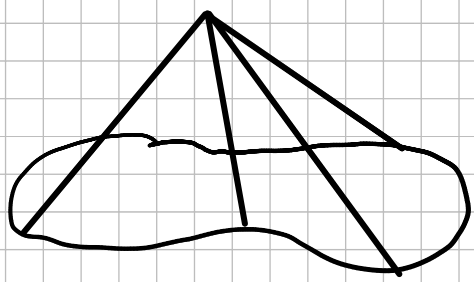
NEPARNE FUNKCIJE NA SIMETRIČNOM DOMENU MOŽE SE IZRAČUNATI DA SU INTEGRALI 0

$$\iint_S \vec{F}\,d\vec{S} = -3B\iint_D dx\,dy = -3B\,P(D) = -3B\pi(4-B^2)$$

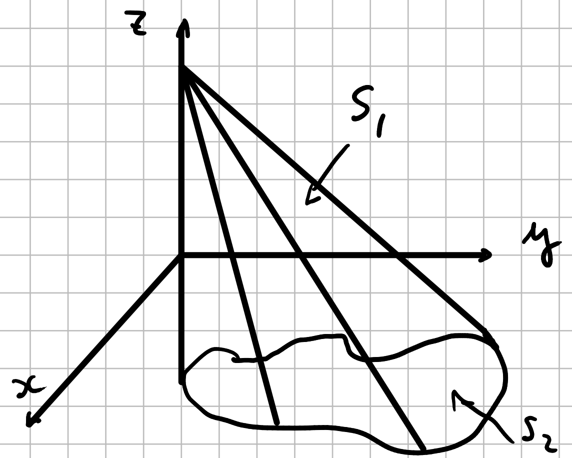
ZAKLJUČAK!

$$\iint_S \vec{F}\, d\vec{S} = \begin{cases} -3B\pi(4-B^2), & 0 > B > -2 \\ 0, & B \leq -2 \end{cases}.$$

⑤ ODREDITI ZAPREMINU KRIVOLINIJSKE KUPE.



POSTAVIMO KUPU U KOORDINATNI SISTEM, TAKO DA JE BAZA KUPE U RAVNI $z=0$, A TEME NA z OSI.



S_1 - OMOTAČ

S_2 - BAZA

$S = S_1 \cup S_2$

T - TELO

NEKA JE $\vec{F} = (x, y, z)$

GAUSOVA FORMULA

$$\iint_S \vec{F}\, d\vec{S} = \iint_S x\, dy\, dz + y\, dz\, dx + z\, dx\, dy =$$

$$= \iiint_T 3\, dx\, dy\, dz = 3 \iiint_T dx\, dy\, dz = 3\, V(T)$$

$$V(T) = \frac{1}{3} \iint_S \vec{F}\, d\vec{S} = \frac{1}{3}\left(\iint_{S_1} \vec{F}\, d\vec{S} + \iint_{S_2} \vec{F}\, d\vec{S} \right)$$

$\underline{S_2}$ $\quad \vec{F} = (x, y, 0)$

PARAMETRIZACIJA $\quad x = x \quad y = y \quad z = 0$

$$d\vec{S} = (0, 0, 1)$$

$$\vec{F} \cdot d\vec{S} = 0$$

$$\iint_{S_2} \vec{F}\, d\vec{S} = 0$$

$\underline{\underline{S_1}}$ TREBA PARAMETRIZOVATI OMOTAČ KUPE

NEKA JE KRIVA KOJA ODREĐUJE BAZU KUPE DATA SA $x = f(u)$ $y = g(u)$

PARAMETRIZACIJA OMOTAČA JE

$$x = -f(u)v \qquad v \in [-1, 0]$$
$$y = -g(u)v$$
$$z = Hv + H$$

$v = 0 \Rightarrow x = 0 \quad y = 0 \quad z = H$

$v = -1 \Rightarrow x = f(u) \quad y = g(u) \quad z = 0$

r - VEKTOR POLOŽAJA

$$\frac{\partial r}{\partial u} = (-f'v, -g'v, 0)$$

$$\frac{\partial r}{\partial v} = (-f, -g, H)$$

$$d\vec{S} = \frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} = (-g'vH, f'vH, f'vg - fg'v)$$

$$\vec{F} = (-fv, -gv, Hv + H)$$

$$\vec{F} \cdot d\vec{S} = \cancel{fg'v^2H} - \cancel{f'gv^2H} + \cancel{Hf'v^2g} + f'gvH - \cancel{Hfg'v^2H} - Hfg'v$$

$$= f'gvH - Hfg'v = Hv(f'g - fg')$$

$$\iint_{S_1} \vec{F}\, d\vec{S} = \iint_D Hv(f'g - fg')\, du\, dv$$

D - BAZA

γ - KRIVA KOJA JE RUB BAZE

$$= H \int_{-1}^{0} v\, dv \int_\gamma f'g - fg'\, du$$

$$= H \frac{1}{2} v^2 \Big|_{-1}^{0} \int_\gamma y\,dx - x\,dy =$$

GRINOVA FORMULA

$$= -H\frac{1}{2} \iint_D -2\,dx\,dy = -\frac{H}{2}(-2)\,P(D) = H\,P(D)$$

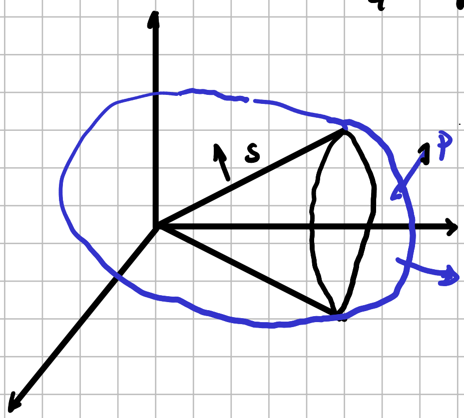
$$V(T) = \frac{1}{3}\iint_{S_1} \vec{F}\, d\vec{S} = \frac{1}{3} H\, P(D)$$

$$\boxed{V(T) = \frac{1}{3} H\, P(D)}$$

⑥ $\iint\limits_S 4\left(\frac{x^2}{4}+y^2+z^2\right)^{2025} yz\,dy\,dz \quad - \quad \left(\frac{x^2}{4}+y^2+z^2\right)^{2025} xy\,dx\,dy$

S : SPOLJNJI DEO POVRŠI $y=\sqrt{x^2+z^2}$

UNUTAR TELA $\frac{x^2}{4}+y^2+z^2 \le 1$



HOĆEMO DA PRIMENIMO GAUSOVU FORMULU, ALI POVRŠ S NIJE GRANICA TELA

DODAJEMO P - DEO ELIPSOIDA

(MOGLI SMO ZATVORITI I U RAVNI, ALI JE OVO LAKŠE, JER NAM SE U INTEGRALU POJAVLJUJE IZRAZ KOJI JE KONSTANTA NA ELIPSOIDU).

$$\iint\limits_{S\cup P} \vec{F}\,d\vec{S} = \iint\limits_S \vec{F}\,d\vec{S} + \iint\limits_P \vec{F}\,d\vec{S} =$$

$$= \iiint\limits_T 4\cdot 2025\left(\frac{x^2}{4}+y^2+z^2\right)^{2024}\frac{x}{2}yz - 2025\left(\frac{x^2}{4}+y^2+z^2\right)^{2024} 2zxy\Big)\,dx\,dy\,dz$$

$$= \iiint\limits_T 0\cdot dx\,dy\,dz = 0$$

$$\Rightarrow \iint\limits_S \vec{F}\,d\vec{S} = -\iint\limits_P \vec{F}\,d\vec{S}$$

NA P VAŽI DA JE $\frac{x^2}{4}+y^2+z^2=1$, PA JE

$$\iint\limits_P \vec{F}\,d\vec{S} = \iint\limits_P 4yz\,dy\,dz - xy\,dx\,dy$$

NA P JE y FUNKCIJA x I z, PA UZIMAMO x I z ZA PARAMETRE.

$$r = (x\ ,\ y(x,z)\ ,\ z)$$

$$\frac{\partial r}{\partial x} = \left(1\ ,\ \frac{\partial y}{\partial x}\ ,\ 0\right)$$

$$\frac{\partial r}{\partial z} = \left(0\ ,\ \frac{\partial y}{\partial z}\ ,\ 1\right)$$

$$\frac{\partial r}{\partial x} \times \frac{\partial r}{\partial z} = \left(\frac{\partial y}{\partial x}\ ,\ -1\ ,\ \frac{\partial y}{\partial z}\right)$$

$$y = \sqrt{1 - \frac{x^2}{4} - z^2}$$

$$\frac{\partial y}{\partial x} = \frac{-\frac{x}{2}}{2\sqrt{1-\frac{x^2}{4}-z^2}} \qquad \frac{\partial y}{\partial z} = \frac{-z}{\sqrt{1-\frac{x^2}{4}-z^2}}$$

$$\iint_P \vec{F}\cdot d\vec{S} = \iint_D \left(4\cancel{\sqrt{1-\frac{x^2}{4}-z^2}}\ z\ \frac{-x}{4\cancel{\sqrt{1-\frac{x^2}{4}-z^2}}} - x\cancel{\sqrt{1-\frac{x^2}{4}\ z^2}}\ \frac{-z}{\cancel{\sqrt{1-\frac{x^2}{4}-z^2}}}\right) dx\, dz$$

$$= \iint_D 0\ dx\, dz = 0$$

$$\Rightarrow \iint_S \vec{F}\cdot d\vec{S} = \boxed{0}$$

GRADIJENT, DIVERGENCIJA, ROTOR

$$f : \mathbb{R}^3 \to \mathbb{R}$$

GRADIJENT - $\text{GRAD } f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$

OPERATOR NABLA $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

$$\text{GRAD } f = \nabla \cdot f = \nabla f$$

MNOŽENJE VEKTORA SKALAROM

$\vec{F} = (P, Q, R) : \mathbb{R}^3 \to \mathbb{R}^3$ VEKTORSKO POLJE

$\operatorname{div} \vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$ – DIVERGENCIJA

$$\operatorname{rot} \vec{F} = \nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

VEKTORSKI PROIZVOD

$$\operatorname{rot} \vec{F} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, -\frac{\partial R}{\partial x} + \frac{\partial P}{\partial z}, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)$$

GRADIJENT : SKALAR $\rightsquigarrow$ VEKTOR

DIVERGENCIJA : VEKTOR $\rightsquigarrow$ SKALAR

ROTOR : VEKTOR $\rightsquigarrow$ VEKTOR

STOKSOVA TEOREMA:

$$\oint_\gamma \vec{F}\, d\vec{r} = \iint_S \operatorname{rot} \vec{F}\, d\vec{S}$$

GAUSOVA TEOREMA:

$$\iint_S \vec{F}\, d\vec{S} = \iiint_T \operatorname{div} \vec{F}\, dx\, dy\, dz$$

7) $\vec{F} = (x, y^2, z^3)$

NAĆI $\operatorname{div} \vec{F}$ U TAČKI $(-2, 4, 5)$

$$P = x \qquad Q = y^2 \qquad R = z^3$$

$$\frac{\partial P}{\partial x} = 1 \qquad \frac{\partial Q}{\partial y} = 2y \qquad \frac{\partial R}{\partial z} = 3z^2$$

$$\operatorname{div} \vec{F} = 1 + 2y + 3z^2$$

$$\operatorname{div} \vec{F}\,(-2, 4, 5) = 1 + 2\cdot 4 + 3\cdot 5^2 = \boxed{84}$$

8) $\vec{F} = f(r)\,\vec{r}$ — ZAVISI OD RASTOJANJA OD $(0,0,0)$

(RADIJALNO POLJE)

POD KOJIM USLOVIMA JE $\operatorname{div}\vec{F} = 0$

$$\vec{F} = f(r)\,\vec{r} = f(r)\,(x, y, z) =$$

$$= (f x + f y + f\cdot z)$$

$$\operatorname{div}\vec{F} = \frac{\partial}{\partial x}(f\cdot x) + \frac{\partial}{\partial y}(f y) + \frac{\partial}{\partial z}(f z)$$

$$= f + x\frac{\partial f}{\partial x} + f + y\frac{\partial f}{\partial y} + f + z\frac{\partial f}{\partial z}$$

$$= 3f + x f'\frac{x}{r} + y f'\frac{y}{r} + z f'\frac{z}{r}$$

$$\frac{\partial}{\partial x} f(r) = \frac{\partial f}{\partial r}\frac{\partial r}{\partial x} = f'\frac{\partial}{\partial x}\left(\sqrt{x^2+y^2+z^2}\right)$$

$$= f'\frac{x}{\sqrt{x^2+y^2+z^2}} = f'\frac{x}{r}$$

$$= 3f + f'\frac{x^2+y^2+z^2}{r} = 3f + f'\frac{r^2}{r}$$

$$= 3f + f' r$$

$$\operatorname{div}\vec{F} = 0 \iff 3f + f' r = 0$$

$$3f = \frac{df}{dr}\, r$$

$$-3\frac{dr}{r} = \frac{df}{f} \;\Big/ \int$$

$$\log f = -3\log r + c \qquad c = \log c_1$$

$$\log f = \log\frac{c_1}{r^3}$$

$$f(r) = \frac{c_1}{r^3}$$

- POLJE KOME JE DIVERGENCIJA NULA NAZIVA SE SOLENOIDNO

(9) $\vec{F} = (z^2, x^2, y^2)$

NAĆI $\operatorname{rot} \vec{F}$ U TAČKI $(1,2,3)$

$$\operatorname{rot} \vec{F} = \nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z^2 & x^2 & y^2 \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial y} y^2 - \frac{\partial}{\partial z} x^2, -\frac{\partial}{\partial x} y^2 + \frac{\partial}{\partial z} z^2, \frac{\partial}{\partial x} x^2 - \frac{\partial}{\partial y} z^2 \right)$$

$$= (2y, 2z, 2x)$$

$$\operatorname{rot} \vec{F} (1,2,3) = (4, 6, 2)$$

(10) ODREDITI ROTOR RADIJALNOG POLJA

$$\vec{F} = f(r)\, \vec{r} = (fx, fy, fz)$$

$$\operatorname{rot} \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xf & yf & zf \end{vmatrix} =$$

$$= \left(z \frac{\partial f}{\partial y} - y \frac{\partial f}{\partial z}, -z \frac{\partial f}{\partial x} + x \frac{\partial f}{\partial z}, y \frac{\partial f}{\partial x} - x \frac{\partial f}{\partial y} \right)$$

$$= \left(z f' \frac{y}{r} - y f' \frac{z}{r}, -z f' \frac{x}{r} + x f' \frac{z}{r}, y f' \frac{x}{r} - x f' \frac{y}{r} \right)$$

$$= (0, 0, 0)$$

DOMAĆI:

(1) $\iint_S (xy + yz + zx)\, dS$

S : DEO KONUSA $z = \sqrt{x^2 + y^2}$ UNUTAR CILINDRA

$$x^2 + y^2 = 2x$$

(2) $$\iint_S z \, dS$$

$S: \quad z = \dfrac{x^2+y^2}{2} \qquad 0 \le z \le 1$

(3) $$\iint_S \frac{dS}{(1+x+y)^2}$$ ← PROMENJEN (OLAKŠAN)

$S = \partial T \qquad T: \ x \geq 0, \ y \geq 0, \ z \geq 0, \ x+y+z \leq 1$

(4) $$\iint_S x \, dy \, dz + y \, dz \, dx + z \, dx \, dy$$

S : UNUTRAŠNJA STRANA KOCKE

$0 \le x \le 2, \quad 0 \le y \le 2, \quad 0 \le z \le 2$

(5) $$\iint_S \vec{F} \cdot d\vec{S}$$

$\vec{F} = (x^2, y^2, z^2)$

$S: \quad z = \sqrt{x^2+y^2} \qquad 0 \le z \le d$ UNUTRAŠNJA STRANA

(6) $$\iint_S \vec{F} \cdot d\vec{S}$$

$\vec{F} = \left(\dfrac{1}{x}, \dfrac{1}{y}, \dfrac{1}{z} \right)$

$S: \quad \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$ ← SPOLJNA STRANA

$a, b, c > 0$

(7) $$\iint_S \vec{F} \cdot d\vec{S}$$

$\vec{F} = (5x + y^2 - z^3, \ x - z^2, \ 2y - z)$

UNUTRAŠNJA STRANA $\quad S = \partial T \qquad T: \ (x-1)^2 + y^2 \le 1 \qquad -x \le z \le x$

(8) $$\iint_S \vec{F} \cdot d\vec{S}$$

$\vec{F} = (-x+y, \ -y + e^z, \ z + x)$

$S = \partial T \qquad T: \ z^2 \geq x^2 + y^2 \qquad 1 \le z \le 2$ — SPOLJNA STRANA