$$f(x) = \frac{\log(4x^2+1)}{4} + x \operatorname{arctg}\left(\frac{2-2x}{4x+1}\right), \quad x = 0.$$

Tejlorov razvoj? $\sum b_n x^n$, $b_n = ?$

$$\operatorname{arctg}\left(\frac{2-2x}{4x+1}\right)$$

$$tg(\alpha-\beta) = \frac{tg\alpha - tg\beta}{1 + tg\alpha \cdot tg\beta}$$

$tg\alpha = 2$, $tg\beta = 2x$

$$\alpha - \beta = \operatorname{arctg}\left(\frac{tg\alpha - tg\beta}{1 + tg\alpha \cdot tg\beta}\right)$$

$$\operatorname{arctg}\left(\frac{2 - 2x}{1 + 2\cdot 2x}\right) = \alpha - \beta = \underbrace{\operatorname{arctg} 2}_{\in\mathbb{R}} - \underbrace{\operatorname{arctg} 2x}_{\text{Razvoj } y\ 0\ ?}$$

$\operatorname{arctg} x = ?$

$g(x) = \operatorname{arctg} x \rightarrow g(0) = 0$

$g'(x) = \frac{1}{1+x^2} \rightarrow g'(0) = 1$ $\quad n = 0$

$g''(x) = -\frac{2x}{(1+x^2)^2} \rightarrow g''(0) = 0$, $\quad g''(x) = -2x \cdot (1+x^2)^{-2}$

Provjera: $g'''(x) = -2x(-2(1+x^2)^{-3}(2x)) - 2(1+x^2)^{-2}$

$g'''(0) = -2$ $\quad \frac{-2}{3!} = -\frac{1}{3}$, $n = 1$

$g^{(4)}(x) = -4x^2(-2\cdot(-3)\cdot(1+x^2)^{-4}\cdot(2x)) - 8x(-2(1+x^2)^{-3}) - 2g''(x)$

$g^{(4)}(0) = 0$ $\quad -8x^3 \cdot 3!(1+x^2)^{-4}$ $\quad -8x\cdot(-2\cdot(1+x^2)^{-3}) - 2g''(x)$

Mogu i na ovaj način

$g^{(5)}(x) = -24x^2 \cdot 3!(1+x^2)^{-4} - 16x^4\cdot(-1)^5\, 4!\,(1+x^2)^{-5}$

$-8\cdot(-2\cdot(1+x^2)^{-3}) \quad - 8x\cdot(-2\cdot(-3)(1+x^2)^{-4}\cdot 2x) - 2g'''(x)$ $\quad$ (3!)

$g^{(5)}(0) = +16 - 2\cdot(-2) = 20 = 5\cdot 4$ $\quad \frac{20}{5!}$ $\quad n = 2$

$g^{(6)}(0) = 0$

$g^{(6)}(x) = -48x\cdot 3!\cdot(1+x^2)^{-4} \quad \underbrace{-48x^3\cdot 3!\,(1+x^2)^{-5}\ldots}\quad \underbrace{-16\cdot 4x^3(\ldots)}$

$$+16\cdot(-3)\cdot(1+x^2)^{-4}\cdot 2x - 32x\cdot 3!\cdot(1+x^2)^{-4} + x^2\cdot(\ldots)$$
$$-2g^{(4)}(x)$$

$$g^{(7)}(x) = -48\cdot 3!\cdot(1+x^2)^{-4} - 6\cdot 16\cdot(1+x^2)^{-4} - 32\cdot 3!(1+x^2)^{-4}$$
$$-2g^{(5)}(x) + x\cdot(\ldots\ldots)$$

$$g^{(7)}(0) = -48\cdot 3! - 3!\cdot 16 - 32\cdot 3! - 2\cdot 20 \qquad 5\cdot 4\cdot 3$$
$$= -4!\cdot(12+4+8) - 2\cdot 20 = \underbrace{-4!\cdot 24 - 2\cdot 20}_{-6!}$$

$$\operatorname{arctg} x = ? \qquad (-1)^n \frac{1}{2n+1} x^{(2n+1)}$$

$$\operatorname{arctg} x = \sum_{n=0}^{+\infty} (-1)^n \frac{1}{2n+1} x^{2n+1}$$

$x=1 \to \sum_{n=0}^{+\infty} \frac{(-1)^n}{2n+1}$ конв. на основу Лајбница условно (не конв. апс.)

$x=-1 \to \sum_{n=0}^{+\infty} \frac{(-1)^n\cdot(-1)^{2n+1}}{2n+1}$ $(-1)^{2n+1}=-1$ $\to$ конв. и за -1

$\Rightarrow$ $D=[-1,1]$

$$\sum_{n=0}^{+\infty} \frac{(-1)^n\cdot(2x)^{2n+1}}{2n+1}, \qquad 2x\in[-1,1]$$
$$x\in[-\tfrac{1}{2},\tfrac{1}{2}]$$

$$f(x) = \frac{\log(4x^2+1)}{4} + x(\operatorname{arctg}2 - \operatorname{arctg}2x)$$

$$\log(1+t) = \sum_{n=1}^{+\infty} \frac{(-1)^{n-1} t^n}{n} = \sum_{n=1}^{+\infty} \frac{(-1)^{n-1} 4^n x^{2n}}{n}, \quad t\in(-1,1]$$
$t = 4x^2$

$4x^2\in(-1,1]$

$x^2\in[0,\frac{1}{4}]$

$x\in[-\frac{1}{2},\frac{1}{2}]$

$$f(x) = \frac{1}{4}\sum_{n=1}^{\infty} \frac{(-1)^{n-1}4^{n-1}x^{2n}}{n} + x\cdot\operatorname{arctg}2 - \sum_{n=0}^{+\infty} \frac{(-1)^n 2^{2n+1}\cdot x^{2(n+1)}}{2n+1}$$
($2^{2n+1} = 4^n\cdot 2$)

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n-1}4^{n-1}x^{2n}}{n} - 2\sum_{n=1}^{\infty} \frac{(-1)^{n-1}4^{n-1}\cdot x^{2n}}{2n-1} + x\cdot\operatorname{arctg}2 =$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n 4^{n-1} x^{2n}}{n(2n-1)} + x \cdot \operatorname{arctg} 2 = \sum_{n=1}^{+\infty} b_n x^n, \quad x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

$\downarrow$

$\frac{1}{n} - \frac{2}{2n-1}$

$\frac{2n-1-2n}{(2n-1)\cdot n}$

$b_1 = \operatorname{arctg} 2$

$b_{2n+1} = 0, \quad n \geq 1$

$b_{2n} = \frac{(-1)^n 4^{n-1}}{n(2n-1)}$

$$= \sum_{n=0}^{\infty} \frac{(-1)^{n+1} 4^n x^{2n+2}}{(2n+1)(n+1)} + x \cdot \operatorname{arctg} 2$$

$(-1)^n \cdot \left(\frac{4}{9}\right)^n = \left(\frac{1}{3}\right)^{2n} \cdot 4^n$

$$\sum_{n=0}^{\infty} \frac{\left(-\frac{4}{9}\right)^n}{(2n+1)(2n+2)} = -9 \sum_{n=0}^{\infty} \frac{(-1)^{n+1} \cdot 4^n \cdot \left(\frac{1}{3}\right)^{2n+2}}{(2n+1)(n+1)} =$$

$\frac{1}{3} \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

$$= -\frac{9}{2}\left(f\left(\frac{1}{3}\right) - \frac{1}{3}\operatorname{arctg} 2\right) = -\frac{9}{2}\left(\frac{\ln\left(4 \cdot \frac{1}{9} + 1\right)}{4} - \frac{1}{3}\operatorname{arctg}\frac{2}{3}\right)$$

$$\sum_{n=1}^{\infty} \sin\frac{n\pi}{3}\left(\sqrt{\frac{(1+n)^3}{n}} + an + b\right)$$

Konv ?

$a, b \in \mathbb{R}$

$\sin(n\alpha)$ ... a_n

$$\sqrt{\frac{(1+n)^3}{n}} + an + b \xrightarrow{?} 0$$

$\geq n$

$$\sqrt{\frac{1+3n+3n^2+n^3}{n}} = \sqrt{n^2 + 3n + 3 + \frac{1}{n}} =$$

$$= n\sqrt{1 + \frac{3}{n} + \frac{3}{n^2} + \frac{1}{n^3}} = n\left(1 + \frac{1}{n}\right)^{+3/2}$$

$(1 + \frac{1}{n})^3$

$$n\left(1 + \frac{1}{n}\right)^{3/2} + an + b \longrightarrow 0 \;?$$

$$1 + \frac{3}{2}\cdot\frac{1}{n} + \binom{\frac{3}{2}}{2}\cdot\frac{1}{n^2} + o\left(\frac{1}{n^2}\right)$$

$\frac{3}{2}\cdot\left(\frac{3}{2} - 1\right)$

$$n\left(1+\frac{3}{2n}+\frac{\frac{3}{2}\cdot\left(\frac{1}{2}\right)}{2}\cdot\frac{1}{n^2}+o\left(\frac{1}{n^2}\right)\right)+an+b$$

$$=\underbrace{n(1+a)+\left(b+\frac{3}{2}\right)+\frac{3}{8n}+o\left(\frac{1}{n}\right)}$$

$a=-1,\ b=-\frac{3}{2}$, $\quad \xrightarrow[n\to+\infty]{} 0$, $\quad \sim +\frac{3}{8n}$, $n\to+\infty$

$\to a\neq 1 \vee b\neq -3/2 \quad \not\to 0$

за $a\neq -1$ или $b\neq -\frac{3}{2}$, $a_n\not\to 0$, $n\to+\infty$

$\Rightarrow$ ред дивергира.

$a=-1,\ b=-\frac{3}{2}$

$$\sum\left|\sin\frac{n\pi}{3}\right|\underbrace{\left|\sqrt{\frac{(1+n)^3}{n}}-n-\frac{3}{2}\right|}_{\sim\frac{3}{8n}}=\sum\frac{1}{2}b_n-\sum\frac{\cos\frac{2n\pi}{3}}{2}b_n$$

$\geq \sin^2\frac{n\pi}{3} = \frac{1-\cos\frac{2n\pi}{3}}{2}$

$\sum\frac{1}{2}b_n$: дивергира поређени

$\sum\frac{\cos\frac{2n\pi}{3}}{2}b_n$: ово b_n ~ ред[illegible] да конв.

$$\sum_{n=1}^{\infty}\sin\frac{n\pi}{3}\underbrace{\left(\sqrt{\frac{(1+n)^3}{n}}-n-\frac{3}{2}\right)}_{b_n}\to \text{конв.}$$

Дирихле:

Д1. $\sum_{n=1}^{\infty}\sin\frac{n\pi}{3}$ има огран. парц. суме ✓

Д2. b_n монотон ? ✓ и да $b_n\to 0$

$b_n\sim+\frac{3}{8n}\xrightarrow[n\to+\infty]{}0$

$$f(x) = \sqrt{\frac{(1+x)^3}{x}} - x - \frac{3}{2}$$

$$f'(x) = \frac{1}{2} \cdot \frac{1}{\sqrt{\frac{(1+x)^3}{x}}} \cdot \frac{3(1+x)^2 \cdot x - (1+x)^3}{x^2} - 1$$

$$= \frac{(1+x)^2 \; (3x - 1 - x)}{2(1+x)^{3/2} \cdot x^{-1/2} \cdot x^2} - 1$$

$$= \frac{(1+x)^{1/2}}{2x^{3/2}} \cdot (2x-1) - 1 \gtrless 0 \;?$$

$$\frac{(1+x)^{1/2}(2x-1)}{2x^{3/2}} \;?\; 1$$

$$(1+x)^{1/2}(2x-1) \;?\; 2x^{3/2} \quad \uparrow^2, \; x>0$$

$$(1+x)\underbrace{(2x-1)^2}_{(4x^2-4x+1)} \;?\; 4x^3$$

$$4x^3 - 4x^2 + x + 4x^2 - 4x + 1 \;?\; 4x^3$$

$$x > 1$$

$$1 \;?\; 3x \qquad 1x \to +\infty$$

$$\boxed{<} \qquad x > 1/3 \checkmark$$

$$\Rightarrow f' < 0, \quad x > 1$$

$$f \downarrow \Rightarrow f(n) \text{ монотонно}$$

b_n монотонн

$$I_n - I_{n-1} = ?$$

$$I_n = \int_0^1 x^n e^{\sqrt{x}}$$

$$\int_0^1 x^{n-1} e^{\sqrt{x}}$$

$$I_{n-1} = \frac{I_n + 4ne}{2n(2n+1)} \quad a_n, \; b_n$$

$$\text{LST}: \quad \frac{I_n + 4ne - (I_{n-1} + 4(n-1)e)}{2n(2n+1) - 2(n-1)(2n-1)}$$