38. г) $$\lim_{x\to 0}\frac{\ln(\cos 5x)-\ln(\cos 7x)}{x^2} = \quad \left\lceil \frac{1-\cos x}{x^2}\to\frac{1}{2},\ x\to 0 \qquad \cos x = 1-\frac{1}{2}x^2+o(x^2),\ x\to 0 \right\rfloor$$

$$= \lim_{x\to 0}\frac{\ln\left(1-\frac{1}{2}(5x)^2+o((5x)^2)\right)-\ln\left(1-\frac{1}{2}(7x)^2+o((7x)^2)\right)}{x^2} = \quad \left\lceil \ln(1+x) = x+o(x),\ x\to 0,\quad \lim_{x\to 0}\frac{\ln(1+x)}{x}=1 \right\rfloor$$

$$= \lim_{x\to 0}\frac{-\frac{1}{2}(5x)^2+\underbrace{o((5x)^2)}_{o(x^2)}+\underbrace{o\left(-\frac{1}{2}(5x)^2+o((5x)^2)\right)}_{o(x^2)}-\left(-\frac{1}{2}(7x)^2+\underbrace{o((7x)^2)}_{o(x^2)}+\underbrace{o\left((7x)^2+o((7x)^2)\right)}_{o(x^2)}\right)}{x^2}$$

$$= \lim_{x\to 0}\frac{-\frac{1}{2}\cdot 25x^2+\frac{1}{2}\cdot 49x^2+o(x^2)}{x^2} = \lim_{x\to 0}\left(12+\overset{\to 0}{o(1)}\right) = 12.$$

$$o\left(\frac{x^2}{x^2}\right) = o(1)$$

39. б) Непрекидност фје

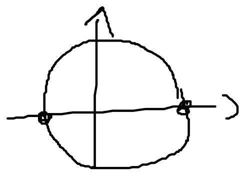
$$f(x) = \begin{cases} \sin \pi x, & x\in\mathbb{Q} \\ 0, & x\in\mathbb{R}\setminus\mathbb{Q} \end{cases}$$

$x\in\mathbb{R}$ $\quad x_n\to x,\ x_n\in\mathbb{Q}$ $\quad \lim_{n\to\infty} f(x_n) = \lim_{n\to\infty}\sin\pi x_n = \sin\pi x$

$\downarrow$ sin непр.

$\quad y_n\to x,\ y_n\in\mathbb{R}\setminus\mathbb{Q}$ $\quad \lim_{n\to\infty} f(y_n) = \lim_{n\to\infty} 0 = 0$

за $\sin\pi x\neq 0$, f има прекид у x.



$\pi x = k\pi,\ k\in\mathbb{Z} \Rightarrow \sin\pi x = 0$

за $\pi x\neq k\pi$ f има прекид у x

$x\neq k,\ k\in\mathbb{Z}$

Хајне: $\forall x_n$ $\quad x_n\to a$ $\quad n\to\infty$ $\quad \lim_{n\to\infty} f(x_n) = b \iff \lim_{x\to a} f(x) = b$.

• $\exists x_n, y_n$: $\quad x_n\to a$ $\quad n\to\infty$ $\quad \lim_{n\to\infty} f(x_n)\neq\lim_{n\to\infty} f(y_n) \iff \nexists \lim_{x\to a} f(x)$.

$x\in\mathbb{Z}$ да ли је f непрекидна у x?

? $\forall\varepsilon>0\ \exists\delta>0\ \forall y\ |y-x|<\delta \Rightarrow |f(y)-f(x)|<\varepsilon$?

$f(x) = \sin\pi x = 0$ ($x\in\mathbb{Z}$) $\quad y\in\mathbb{R}\setminus\mathbb{Q}$ $\quad |f(y)-f(x)| = |0-0| = 0<\varepsilon$.

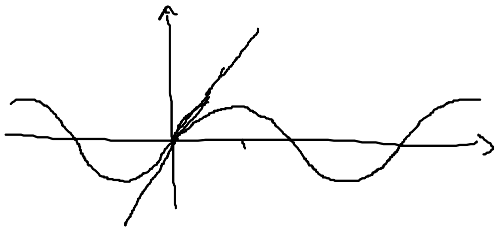
$y \in \mathbb{Q}$ $\quad |f(y)-f(x)| = |\sin \pi y - \sin \pi x| = \underbrace{\left|2\sin\left(\pi \frac{y-x}{2}\right)\cdot \cos\left(\frac{y+x}{2}\pi\right)\right|} < \varepsilon$

ово хоћемо за $|x-y|<\delta$

$$\left|2\sin\left(\pi \frac{y-x}{2}\right)\cos\left(\frac{y+x}{2}\pi\right)\right| = 2\left|\sin\left(\pi\frac{y-x}{2}\right)\right| \underbrace{\left|\cos \pi\cdot\frac{y+x}{2}\right|}_{\le 1}$$

$$\le 2\left|\sin \pi\cdot\frac{y-x}{2}\right| \le 2\cdot\left|\pi\frac{(y-x)}{2}\right| = \pi\cdot|y-x| < \varepsilon$$

$|\sin t| \le |t|$



$$|y-x| < \frac{\varepsilon}{\pi} \Rightarrow \boxed{\delta = \frac{\varepsilon}{\pi}}$$

$\Rightarrow f$ је непрекидна у x за $x \in \mathbb{Z}$.

41. б)

$f(x) = tg\, x - \sin x \quad , x \to 0$

$\sin x = x + o(x)$

$$tg\, x = \frac{\sin x}{\cos x} = \frac{x+o(x)}{1-\frac{1}{2}x^2+o(x^2)} = (x+o(x))\Big(1\underbrace{-\frac{1}{2}x^2+o(x^2)}_{t}\Big)^{-1} =$$

$\left[(1+t)^\alpha = 1+\alpha t + o(t) \right.$, $t \to 0$

$$= (x+o(x))\left(1+(-1)\cdot\left(-\frac{1}{2}x^2+o(x^2)\right) + o\left(-\frac{1}{2}x^2+o(x^2)\right)\right) =$$

$o(-\frac{1}{2}x^2) + o(o(x^2))$

$$= (x+o(x))\Big(1+\frac{1}{2}x^2 \underbrace{- o(x^2) + o\left(-\frac{1}{2}x^2+o(x^2)\right)}_{o(x^2)}\Big) =$$

$$= (x+o(x))\left(1+\frac{1}{2}x^2+o(x^2)\right)$$

$$tg\, x - \sin x = \sin x\left(1+\frac{1}{2}x^2+o(x^2)\right) - \sin x$$

$$= \sin x\left(1+\frac{1}{2}x^2+o(x^2) - 1\right) =$$

$$= (x+o(x))\left(\frac{1}{2}x^2+o(x^2)\right) = \frac{1}{2}x^3 + \underbrace{\frac{1}{2}x^2 o(x)}_{o(x^3)} + \underbrace{x\cdot o(x^2)}_{o(x^3)} + \underbrace{o(x)\cdot o(x^2)}_{o(x^3)}$$

$$= \frac{1}{2}x^3 + o(x^3) \quad , x \to 0$$

$tg\, x - \sin x \sim \frac{1}{2}x^3$

$$tg\, x - \sin x = (x+o(x))\left(1+\frac{1}{2}x^2+o(x^2)\right) - (x+o(x))$$

$$x+o(x) + \frac{1}{2}x^3 + \dots \quad - \cancel{x} + o(x) = o(x) + \underbrace{\frac{1}{2}x^3 + o(x^3)}_{\parallel} = o(x)$$

$$x\cdot\Big(\underbrace{\frac{1}{2}x^2+o(x^2)}_{\substack{\longrightarrow 0 \\ x\to 0}}\Big)$$

$tg\, x - \sin x = o(x)$ .... нисмо добили одгов. полином ...

$f = o(g)$, $x \to a$ $\lim \alpha = 0$ $f = \alpha g$ $\Leftarrow$ $\alpha = \gamma - 1$

, $\lim \frac{f}{g} = 0$,

$f \sim g$ γ $\lim \frac{f}{g} = 1$ $\Rightarrow$ $o(f) = o(g)$ $\frac{f}{g} = \gamma(x)$ $\alpha \to 0$, $x \to a$

$f = g + o(g)$ $f = \gamma g = g + (\gamma - 1) \cdot g = g + o(g)$, $x \to a$

$o(f) = o(g) \not\Rightarrow f \sim g$,

$f = 2x$, $g = x$ $f \not\sim g$ $\lim \frac{f}{g} = 2$ $o(2x) = o(x)$.

$f = O(g)$ $\lim \frac{f}{g} \in \mathbb{R} \setminus \{0\} \Leftrightarrow o(f) = o(g)$

$o(1) \to 0$, $x \to a$ $o(c \cdot 1) = o(1)$

$o(c) \longrightarrow 0$