30. $\lim\limits_{n\to\infty} \frac{(1\cdot 1!)^2+(2\cdot 2!)^2+\dots+(n\cdot n!)^2}{((n+1)!)^2} = \lim\limits_{n\to\infty} \frac{a_n}{b_n} \overset{*}{\underset{ШТ}{=}} \lim\limits_{n\to\infty} \frac{a_{n+1}-a_n}{b_{n+1}-b_n} =$

$b_n = ((n+1)!)^2$

$\lim\limits_{n\to\infty} b_n = +\infty$, $b_n \uparrow$

$= \lim\limits_{n\to\infty} \frac{((n+1)\cdot(n+1)!)^2}{((n+2)!)^2-((n+1)!)^2} = \lim\limits_{n\to\infty} \frac{(n+1)^2\cdot\cancel{((n+1)!)^2}}{\cancel{((n+1)!)^2}((n+2)^2-1)} =$

$= \lim\limits_{n\to\infty} \frac{(n+1)^2}{(n+2)^2-1} = 1$.

35. (x_n) монотон

$\lim\limits_{n\to\infty}(2x_{n+1}-x_n) = x$, $x\in\mathbb{R}$

(x_n) конвергира ?

1) П. $x_n \uparrow$ $\quad x_{n+1} > x_n$

x_n ограничен ?

$2x_{n+1}-x_n = x_{n+1} + \underbrace{(x_{n+1}-x_n)}_{>0}$

ако x_n није ограничен онда $\lim\limits_{n\to\infty} x_n = +\infty \Rightarrow \forall M>0 \ \exists n_0 \ \forall n\geq n_0 \quad x_n > M$

$\Rightarrow x_{n+1} + \underbrace{(x_{n+1}-x_n)}_{>0} > M$

$\Rightarrow \lim\limits_{n\to\infty}(2x_{n+1}-x_n) = +\infty \ \not\!\!\!\!\!\!\lightning \ x\in\mathbb{R}$

$\Rightarrow x_n$ мора бити огран.

пп. $x_n \downarrow$

$\left.\begin{array}{l} 2x_{n+1}-x_n = x_{n+1} + \underbrace{(x_{n+1}-x_n)}_{<0} \\ x_n \text{ није огран} \Rightarrow \lim\limits_{n\to\infty} x_{n+1} = -\infty \end{array}\right\} \Rightarrow \lim\limits_{n\to\infty}(2x_{n+1}-x_n) = -\infty \ \lightning \ x\in\mathbb{R}$

$\Rightarrow x_n$ огран.

$\Rightarrow x_n$ конвергира! $\quad \lim\limits_{n\to\infty} x_n = y$

$\Rightarrow x = \lim\limits_{n\to\infty}(2x_{n+1}-x_n) = 2y-y = y$.

33. $\lim_{n\to\infty} a_n = a$, $\alpha \in (0,1)$, $\lim_{n\to\infty} \alpha^n = 0$ $\lim_{n\to\infty} \frac{1^n}{\alpha^n} = +\infty$, $\lim_{n\to\infty} \left(\frac{1}{\alpha}\right)^n = +\infty$

$$\lim_{n\to\infty} a_n + \alpha a_{n-1} + \dots + \alpha^{n-2} a_2 + \alpha^{n-1} a_1 = \lim_{n\to\infty} a_n + \frac{1}{\frac{1}{\alpha}} \cdot a_{n-1} + \frac{1}{\left(\frac{1}{\alpha}\right)^2} a_{n-2} + \dots + \frac{1}{\left(\frac{1}{\alpha}\right)^{n-1}} a_1$$

$$= \lim_{n\to\infty} \frac{\left(\frac{1}{\alpha}\right)^{n-1} \cdot a_n + \left(\frac{1}{\alpha}\right)^{n-2} \cdot a_{n-1} + \left(\frac{1}{\alpha}\right)^{n-3} a_{n-2} + \dots + a_1}{\left(\frac{1}{\alpha}\right)^{n-1}} \underset{\text{ШТ}}{\overset{(*)}{=}} \lim_{n\to\infty} \frac{\left(\frac{1}{\alpha}\right)^n \cdot a_{n+1}}{\left(\frac{1}{\alpha}\right)^n - \left(\frac{1}{\alpha}\right)^{n-1}} =$$

$b_n = \left(\frac{1}{\alpha}\right)^{n-1}$, $\lim_{n\to\infty} b_n = +\infty$

$b_{n+1} = \frac{1}{\alpha} \cdot b_n > b_n$ ✓

$$= \lim_{n\to\infty} \frac{\frac{1}{\alpha} \cdot a_{n+1}}{\frac{1}{\alpha} - 1} = \lim_{n\to\infty} \frac{a_{n+1}}{1-\alpha} = \frac{a}{1-\alpha}.$$