15) $a, b, c, d > 0$

$$\underbrace{\frac{a}{b+c+d}}_{a_1^2} + \underbrace{\frac{b}{c+d+a}}_{a_2^2} + \underbrace{\frac{c}{d+a+b}}_{a_3^2} + \underbrace{\frac{d}{a+b+c}}_{a_4^2} \geq \frac{4}{3}$$

за $b_1 = \sqrt{a} \cdot \sqrt{b+c+d}$

$$a_1 \cdot b_1 = \sqrt{\frac{a}{b+c+d}} \cdot \sqrt{a} \cdot \sqrt{b+c+d} = a$$

$$\underset{\substack{\parallel \\ (a(b+c+d)+b(a+c+d)+\dots)}}{A \cdot B} \geq (a+b+c+d)^2$$

$= ab+ac+cd+ad+bd+cb)$

$$? \quad \underset{\substack{\parallel \\ a^2+b^2+c^2+d^2+B}}{(a+b+c+d)^2} \geq \frac{4}{3} \cdot B = \frac{8}{3}(ab+ac+cd+ad+bd+cb)$$

$$a^2+b^2+c^2+d^2+B \geq \frac{4}{3} B$$

$$3(a^2+b^2+c^2+d^2) \geq B = 2(ab+ac+ad+bc+bd+cd)$$

$$\underbrace{(a^2+b^2)}_{\geq 2ab} + \underbrace{(a^2+c^2)}_{\geq 2ac} + \underbrace{(a^2+d^2)}_{\geq 2ad} + (b^2+c^2) + (b^2+d^2) + (c^2+d^2) \geq B$$

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$$A \cdot B \geq (a+b+c+d)^2 \geq 4/3\, B \Rightarrow \boxed{A \geq 4/3}$$

16) $$A_3 = \left\{ \underbrace{\frac{n^3 (m+1)^4}{8(m^3+2n^3)\cdot(-m)^4}} \;\middle|\; m, n \in \mathbb{N} \right\}$$

$$\left(1+\frac{1}{m}\right)^m \xrightarrow[m \to +\infty]{} e$$

$\inf A_3 = ?$

$\sup A_3 = ?$

$$b_{m,n} = \frac{1}{8} \cdot \frac{n^3}{2n^3+m^3} \cdot (-1)^m \cdot \left(1+\frac{1}{m}\right)^{m} {}^{<e}$$

за n фиксирано член $b_{m,n} \to 0$, $m \to +\infty$

$$A_3 = \bigcup_{m\in\mathbb{N}} A_m$$

$$A_m = \left\{ b_{m,n} \mid n\in\mathbb{N} \right\} = \left\{ \frac{1}{8} \frac{n^3}{2n^3+m^2} \cdot (-1)^m \left(1+\frac{1}{m}\right)^m \mid n\in\mathbb{N} \right\}$$

m – парно

$$A_m = \Bigg\{ \underbrace{\frac{1}{8}\left(\frac{1}{2} - \frac{m^2}{2n^3+m^2}\right)}_{\substack{\geq \\ 0}\quad b_{m,n}\uparrow} \cdot \left(1+\frac{1}{m}\right)^m \mid n\in\mathbb{N} \Bigg\}$$

$$\Rightarrow \inf A_m = \min A_m = b_{m,1} = \frac{1}{8}\cdot\frac{1}{m^2+2}\cdot\left(1+\frac{1}{m}\right)^m$$

$$\sup A_m = \frac{1}{16}\cdot\left(1+\frac{1}{m}\right)^m$$

m – непарно

$$A_m = \left\{ -\frac{1}{8}\left(\frac{1}{2} - \frac{m^2}{2n^3+m^2}\right)\cdot\left(1+\frac{1}{m}\right)^m \mid n\in\mathbb{N} \right\}$$

$$\left(1+\frac{1}{m}\right)^m \uparrow e$$

$$\sup A_m = -\frac{1}{8}\cdot\frac{1}{m^2+2}\cdot\left(1+\frac{1}{m}\right)^m$$

$$\inf A_m = -\frac{1}{16}\cdot\left(1+\frac{1}{m}\right)^m$$

$$\sup A = \sup \bigcup_{m=1}^{\infty} A_m = \sup\{\sup A_m\} = \sup\left\{\frac{1}{16}\cdot\left(1+\frac{1}{m}\right)^m \mid m\in\mathbb{N}\right\} = \frac{e}{16}$$

$$\inf A = \inf \bigcup_{m=1}^{\infty} A_m = \inf\{\inf A_m\} = -\frac{1}{16}\cdot e$$

Треба показати да је ово заиста sup / inf.