a) $a_n = \dfrac{\underbrace{\operatorname{arctg} n^{\alpha}}_{\to \frac{\pi}{2}} - \frac{\pi}{2}\cosh\frac{1}{\sqrt{n^3}} + \left(\frac{n^3+2}{n^3+3}\right)^3 - \left(\cos\frac{1}{n}\right)^{\sin\frac{1}{n+2}}}{n^{\beta}}$

$\alpha > 0,\ \beta \in \mathbb{R}$

$\underline{\cosh x} = \dfrac{e^x + e^{-x}}{2}$

$\cosh\frac{1}{\sqrt{n^3}} = \cosh\frac{1}{n^{3/2}} = \dfrac{e^{\frac{1}{n^{3/2}}} + e^{-\frac{1}{n^{3/2}}}}{2} = \Big\lceil e^t = 1 + t + \frac{t^2}{2} + \frac{t^3}{3!} + \frac{t^4}{4!} + \dots + \frac{t^n}{n!} + o(t^n),\ t \to 0 \Big\rfloor$

$= \dfrac{1 + \cancel{\frac{1}{n^{3/2}}} + \frac{\frac{1}{n^3}}{2} + \cancel{\frac{\frac{1}{n^{9/2}}}{3!}} + \frac{\frac{1}{n^6}}{4!} + o\left(\frac{1}{n^6}\right) + 1 - \cancel{\frac{1}{n^{3/2}}} + \frac{\frac{1}{n^3}}{2} - \cancel{\frac{\frac{1}{n^{9/2}}}{3!}} + \frac{\frac{1}{n^6}}{4!} + o\left(\frac{1}{n^6}\right)}{2}$

$= \underline{1 + \frac{1}{2n^3} + \frac{1}{4!\,n^6} + o\left(\frac{1}{n^6}\right)}$

$\left(\frac{n^3+2}{n^3+3}\right)^3 = \left(1 - \frac{1}{n^3+3}\right)^3 = 1 - \frac{3}{n^3+3} + \frac{3}{(n^3+3)^2} - \frac{1}{(n^3+3)^3} = 1 - \frac{3}{n^3} + \frac{9}{n^6} + \frac{3}{n^6} + o\left(\frac{1}{n^6}\right)$

$\left(\frac{1}{n^3+3}\right)^k = (3+n^3)^{-k} = n^{-3k}\cdot\Big(1 + \underbrace{\frac{3}{n^3}}_{\to 0}\Big)^{-k} = \Big\lceil (1+t)^{\alpha} = 1 + \binom{\alpha}{1}t + \binom{\alpha}{2}t^2 + \dots,\ t \to 0 \Big\rfloor$

$k = 1 \qquad \frac{1}{n^3+3} = \frac{1}{n^3}\cdot\left(1 + \frac{3}{n^3}\right)^{-1} = \frac{1}{n^3}\left(1 - \underline{\underline{\frac{3}{n^3}}} + o\left(\frac{1}{n^3}\right)\right)$

$k = 2 \qquad \frac{1}{(n^3+3)^2} \sim \frac{1}{n^6} + o\left(\frac{1}{n^6}\right)$

$\left(\cos\frac{1}{n}\right)^{\sin\frac{1}{n+2}} = e^{\sin\frac{1}{n+2}\ln\left(\cos\frac{1}{n}\right)} = \Big\lceil \cos t = 1 - \frac{t^2}{2} + \frac{t^4}{4!} - \frac{t^6}{6!} + o(t^7),\ t \to 0$

$\sin t = t - \frac{t^3}{3!} + \frac{t^5}{5!} + o(t^6),\ t \to 0 \Big\rfloor =$

$= e^{\left(\frac{1}{n+2} - \frac{1}{6\cdot(n+2)^3} + \frac{1}{120(n+2)^5} + o\left(\frac{1}{(n+2)^6}\right)\right)\ln\Big(1 \underbrace{- \frac{1}{2n^2} + \frac{1}{24n^4} - \frac{1}{720n^6}}_{\sim\frac{1}{n^2}} + o\left(\frac{1}{n^7}\right)\Big)}$ " t

$= \Big\lceil \ln(1+t) = \underbrace{t - \frac{t^2}{2} + \frac{t^3}{3}} - \frac{t^4}{4} + \frac{t^5}{5} - \frac{t^6}{6} + o(t^6) \Big\rfloor =$

$= e^{\left(\frac{1}{n+2} - \frac{1}{6(n+2)^3} + \frac{1}{120(n+2)^5} + o\left(\frac{1}{(n+2)^6}\right)\right)\left(-\frac{1}{2n^2} + \frac{1}{24n^4} - \frac{1}{720n^6} + o\left(\frac{1}{n^7}\right) - \frac{1}{2}\left(-\frac{1}{2n^2} + \frac{1}{24n^4} - \frac{1}{720n^6} + o\left(\frac{1}{n^7}\right)\right)^2 + \frac{1}{3}\left(-\frac{1}{2n^2} + \frac{1}{24n^4} - \frac{1}{720n^6} + o\left(\frac{1}{n^7}\right)\right)^3 + o\left(\left(\text{—}\,\|\,\text{—} \right)^3\right)\right)}$

$$= e^{\left(\frac{1}{n+2} - \frac{1}{6(n+2)^3} + \frac{1}{120(n+2)^5} + o\left(\frac{1}{n^6}\right)\right)\left(-\frac{1}{2n^2} + \frac{1}{24n^4} - \frac{1}{720n^6} + o\left(\frac{1}{n^7}\right) - \frac{1}{2}\left(\frac{1}{4n^4} - \frac{\cancel{2}}{\cancel{2}\cdot 24\cdot n^6} + o\left(\frac{1}{n^6}\right)\right) + \frac{1}{3}\left(-\frac{1}{8n^6} + o\left(\frac{1}{n^6}\right)\right) + o\left(\frac{1}{n^6}\right)\right)}$$

$$= e^{\left(\frac{1}{n+2} - \frac{1}{6(n+2)^3} + \frac{1}{120(n+2)^5} + o\left(\frac{1}{n^6}\right)\right)\left(-\underline{\underline{\frac{1}{2n^2}}} - \frac{1}{12}\cdot\frac{1}{n^4} + \underbrace{\underline{\underline{\frac{1}{n^6}}}\left(-\frac{1}{720} + \frac{1}{48} - \frac{1}{24}\right) + o\left(\frac{1}{n^6}\right)}_{o\left(\frac{1}{n^5}\right) = \frac{1}{n^2} o\left(\frac{1}{n^3}\right)}\right)}$$

$$(n+2)^{-1} = n^{-1}\cdot\left(1+\frac{2}{n}\right)^{-1} = \frac{1}{n}\left(1+\frac{2}{n}\right)^{-1} = \frac{1}{n}\left(1 - \frac{2}{n} + \frac{4}{n^2} - \frac{8}{n^3} + o\left(\frac{1}{n^3}\right)\right)$$

$$(n+2)^{-3} = \frac{1}{n^3}\left(1+\frac{2}{n}\right)^{-3} = \frac{1}{n^3}\left(1 + \binom{-3}{1}\cdot\frac{2}{n} + o\left(\frac{1}{n}\right)\right) \qquad \binom{-3}{1} = \frac{-3}{1!} = -3$$

$$(n+2)^{-5} = \frac{1}{n^5}\left(1+\frac{2}{n}\right)^{-5} = o\left(\frac{1}{n^4}\right)$$

$$= e^{\frac{1}{n}\left(1 - \frac{2}{n} + \frac{4}{n^2} - \frac{8}{n^3} + o\left(\frac{1}{n^3}\right)\right) + \frac{1}{n^2} - \frac{6}{n^3} + o\left(\frac{1}{n^3}\right)\Big)\underline{\underline{\frac{1}{n^2}}}\left(-\frac{1}{2} - \frac{1}{12n^2} + o\left(\frac{1}{n^3}\right)\right)}$$

$$= e^{\frac{1}{n^3}\left(-\frac{1}{2} + \frac{1}{n} - \frac{2}{n^2} + \frac{4}{n^3} - \frac{1}{2n^2} + \frac{3}{n^3} - \frac{1}{12n^2} + \frac{1}{6n^3} + o\left(\frac{1}{n^3}\right)\right)}$$

$$= e^{\frac{1}{n^3}\left(-\frac{1}{2} + \frac{1}{n} - \frac{31}{12n^2} + \frac{43}{6n^3} + o\left(\frac{1}{n^3}\right)\right)} =$$

$$-2 - \frac{1}{2} - \frac{1}{12} = -\frac{24+6+1}{12} = -\frac{31}{12}$$

$$4 + 3 + \frac{1}{6} = \frac{24+18+1}{6} = \frac{43}{6}$$

$$= \left\lceil e^t = 1 + t + \frac{t^2}{2} + o(t^2) \right\rfloor = 1 + \frac{1}{n^3}\left(-\frac{1}{2} + \frac{1}{n} - \frac{31}{12n^2} + \frac{43}{6n^3} + o\left(\frac{1}{n^3}\right)\right)$$

$$+ \frac{1}{2n^6}\left(-\frac{1}{2} + \frac{1}{n} - \dots\right)^2$$

$$+ o\left(\frac{1}{n^6}\right)$$

$$= 1 - \frac{1}{2n^3} + \frac{1}{n^4} - \frac{31}{12n^5} + \frac{43}{6n^6} + \frac{1}{8n^6} + o\left(\frac{1}{n^6}\right)$$

$$\frac{\pi}{2} - \operatorname{arctg} n^{\alpha} \qquad ?$$

$$-\frac{\pi}{2}\left(1 + \frac{1}{2n^3} + \frac{1}{24n^6} + o\left(\frac{1}{n^6}\right)\right) + \cancel{1} - \frac{3}{n^3} + \frac{12}{n^6} + o\left(\frac{1}{n^6}\right) -$$

$$-\left(\cancel{1} - \frac{1}{2n^3} + \frac{1}{n^4} - \frac{31}{12n^5} + \frac{43}{6n^6} + \frac{1}{8n^6} + o\left(\frac{1}{n^6}\right)\right)$$

$$= -\frac{\pi}{2} - \frac{\pi}{4}\frac{1}{n^3} - \frac{\pi}{48}\cdot\frac{1}{n^6} - \frac{3}{n^3} + \frac{1}{2n^3} - \frac{1}{n^4} + \dots + o\left(\frac{1}{n^6}\right)$$

$$= \underbrace{-\frac{\pi}{2} + \left(-\frac{\pi}{4} - 3 + \frac{1}{2}\right)\frac{1}{n^3} + o\left(\frac{1}{n^3}\right)}$$

$$\frac{\pi}{2} - \operatorname{arctg} n^\alpha = \underline{n^{?}} \to 0$$

$\alpha > 0,\ n^\alpha \to +\infty$

$$\operatorname{arctg} n^\alpha = \frac{\pi}{2} - \frac{1}{n^\alpha} - \frac{1}{3n^{3\alpha}} + o\left(\frac{1}{n^{3\alpha}}\right)$$

$$\operatorname{arctg} x \to \frac{1}{1+x^2} = (1+x^2)^{-1}$$

$$\operatorname{arctg} x + \operatorname{arctg}\frac{1}{x} = \frac{\pi}{2},\quad x > 0$$

$$\to -(1+x^2)^{-2}\cdot 2x$$

$$x \to +\infty,\ \operatorname{arctg} x = \frac{\pi}{2} - \operatorname{arctg}\frac{1}{x} = \frac{\pi}{2} - \left(\frac{1}{x} - \frac{1}{3x^3} + o\left(\frac{1}{x^3}\right)\right)$$

$$\to 2(1+x^2)^{-3}\cdot 4x^2 - 2(1+x^2)^{-2}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} n^{-\beta}\left(\cancel{\frac{\pi}{2}} - \underline{\frac{1}{n^\alpha}} + o\left(\frac{1}{n^\alpha}\right) - \cancel{\frac{\pi}{2}} + \underline{\left(-\frac{\pi}{4} - \frac{5}{2}\right)\frac{1}{n^3}} + o\left(\frac{1}{n^3}\right)\right)$$

$$= \begin{cases} \lim\limits_{n\to\infty} \left(-\frac{\pi}{4} - \frac{7}{2}\right) n^{-\beta-3} + o(n^{-\beta-3}), & \alpha = 3 \\ \lim\limits_{n\to\infty} \left(-\frac{\pi}{4} - \frac{5}{2}\right) n^{-\beta-3} + o(n^{-\beta-3}), & \alpha > 3 \\ \lim\limits_{n\to\infty} -n^{-\beta-\alpha} + o(n^{-\beta-\alpha}) & \alpha < 3 \end{cases}$$

I случай $\alpha = 3$:

I.1° $\beta = -3 \Rightarrow \lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \left(-\frac{\pi}{4} - \frac{7}{2}\right) n^0 + o(n^0) = -\frac{\pi}{4} - \frac{7}{2}$

I.2° $\beta < -3 \Rightarrow \lim\limits_{n\to\infty} a_n = \left(-\frac{\pi}{4} - \frac{7}{2}\right)\lim\limits_{n\to\infty} n^{-\beta-3} = -\infty$

I.3° $\beta > -3 \Rightarrow \lim\limits_{n\to\infty} a_n = \left(-\frac{\pi}{4} - \frac{7}{2}\right)\lim\limits_{n\to\infty} n^{\overset{<0}{-\beta-3}} = 0$

II случай $\alpha > 3$:

II.1° $\beta = -3 \Rightarrow \lim\limits_{n\to+\infty} a_n = -\frac{\pi}{4} - \frac{5}{2}$

II.2° $\beta < -3 \Rightarrow \lim\limits_{n\to\infty} a_n = -\infty$

II.3° $\beta > -3 \Rightarrow \lim\limits_{n\to\infty} a_n = 0$

III случай $\alpha < 3$:

III.1° $\beta = -\alpha \Rightarrow \lim\limits_{n\to\infty} a_n = -1$

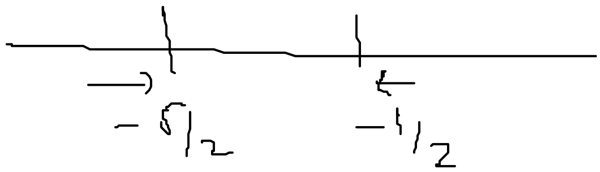
III.2° $\beta < -\alpha \Rightarrow \lim\limits_{n\to\infty} a_n = -\infty$

III.3° $\beta > -\alpha \Rightarrow \lim\limits_{n\to\infty} a_n = 0.$

52. b)

$$f(x) = \begin{cases} ax + \sqrt[3]{2x-1}, & x \in \left[-\frac{5}{2}, -\frac{1}{2}\right] \\ c \arcsin \frac{2}{2x+3}, & x \notin [-5/2, -1/2] \end{cases}$$

f деф ?



$$\left|\frac{2}{2x+3}\right| \leq 1 \qquad 2 \leq |2x+3|$$

$$x \neq -\frac{2}{3}$$

$$2x+3 \geq 2 \quad \vee \quad 2x+3 \leq -2$$

$$x \geq -\frac{1}{2} \quad \vee \quad x \leq -\frac{5}{2}$$

f јесте добро деф.

$$\lim_{x \to -\frac{5}{2}^-} c \cdot \arcsin \frac{2}{2x+3} = c \arcsin \frac{2}{-5+3} = c \arcsin(-1) = -\frac{\pi}{2} c.$$

$$= f\left(-\frac{5}{2}\right) = -\frac{5}{2} a + \sqrt[3]{-5-1}$$

$$\Rightarrow \quad -\frac{\pi}{2} c = -\frac{5}{2} a - \sqrt[3]{6}$$

$$\lim_{x \to -\frac{1}{2}^+} f(x) = c \arcsin 1 = c \frac{\pi}{2} = -\frac{a}{2} + \sqrt[3]{-2}$$

$$-3a - \sqrt[3]{6} - \sqrt[3]{2} = 0$$

$$a = \frac{-\sqrt[3]{6} - \sqrt[3]{2}}{3}, \quad c = \dots \in \mathbb{R}$$

$$f'(x) = \begin{cases} a + \frac{1}{3}(2x-1)^{-2/3} \cdot 2, & x \in \left(-5/2, -\frac{1}{2}\right) \\ c \cdot \frac{1}{\sqrt{1 - \frac{4}{(2x+3)^2}}} \cdot (-2) \cdot \frac{1}{(2x+3)^2} \cdot 2, & x \notin \left[-\frac{5}{2}, -\frac{1}{2}\right] \setminus \{-\frac{7}{2}, \frac{1}{2} \ldots \end{cases}$$

$x \notin \left[-\frac{5}{2}, -\frac{1}{2}\right] \setminus \left\{-\frac{7}{2}, \frac{1}{2}\right\}$ $f'(x) = \dfrac{-4c}{\sqrt{(2x+7)(2x-1)}\;|2x+3|}$

$-\frac{7}{2}$: $f'\left(-\frac{7}{2}\right) = \lim\limits_{h \to 0} \dfrac{f\left(-\frac{7}{2}+h\right) - f\left(-\frac{7}{2}\right)}{h}$ $\downarrow$

$x = -\frac{7}{2}$

$x = \frac{1}{2}$

$= \lim\limits_{h \to 0} \dfrac{c \arcsin \frac{2}{-7+2h+3} - c \arcsin \frac{2}{-4}}{h}$

$= c \lim\limits_{h \to 0} \dfrac{\arcsin\left(\frac{2}{-4+2h}\right) - \left(-\frac{\pi}{6}\right)}{h} =$ ☺

$\arcsin\left(\dfrac{2}{-4+2h}\right) = \arcsin\left(2 \cdot (-4+2h)^{-1}\right)$

$= \arcsin\left(-\frac{1}{2}\left(1 - \frac{h}{2}\right)^{-1}\right)$

$1 - \left(-\frac{h}{2}\right) + o(h)$

$\dfrac{1}{\sqrt{1-x^2}}$, $x = -\frac{1}{2}$ $\qquad = \arcsin\left(-\frac{1}{2} - \frac{h}{4} + o(h)\right)$

$f'\left(-\frac{1}{2}\right) = \dfrac{1}{\sqrt{1-\frac{1}{4}}} = \dfrac{2}{\sqrt{3}}$ $\qquad \overset{*}{=} \arcsin\left(-\frac{1}{2}\right) + \frac{2}{\sqrt{3}} \cdot \left(\cancel{-\frac{1}{2}} - \frac{h}{4} + o(h) - \left(\cancel{-\frac{1}{2}}\right)\right)$

Тејлоров развој $y - \frac{1}{2}$ $\qquad + o\left(\quad\right)$

$= -\frac{\pi}{6} - \frac{1}{2\sqrt{3}}h + o(h)$

☺ $= c \lim\limits_{h \to 0} \dfrac{\cancel{-\frac{\pi}{6}} - \frac{1}{2\sqrt{3}}h + o(h) + \cancel{\frac{\pi}{6}}}{h} = -\dfrac{1}{2\sqrt{3}}c$

$\frac{1}{2}$: $f'\left(\frac{1}{2}\right) = c \lim\limits_{h \to 0} \dfrac{\arcsin\left(\frac{2}{4+2h}\right) - \arcsin \frac{1}{2}}{h} = \ldots$ за вежбу

$-\frac{5}{2}: \quad f'_{+}\left(-\frac{5}{2}\right), \quad f'_{-}\left(-\frac{5}{2}\right)$, " = "

$-\frac{1}{2}:$ $\leftarrow$

$\Bigg\{$ ЗА
вешбу