55. $\left.\begin{matrix} f\in D(\mathbb{R}) \\ 0<a<b \end{matrix}\right\} \overset{?}{\Rightarrow} \exists \xi\in(a,b) \quad \frac{b-a}{2}\frac{f'(\xi)}{\xi} = \frac{f(b)-f(a)}{b+a}$.

Лагранж: $f\in C[a,b]\cap D(a,b)$

$\exists c : \frac{f(b)-f(a)}{b-a} = f'(c)$

Дарбу: $c,d\in(a,b)$

$\forall \gamma\in (f'(c), f'(d)) \quad \exists \xi\in(c,d) \quad f'(\xi)=\gamma$

Коши: $f,g\in C[a,b]\cap D(a,b) \quad g'(x)\neq 0, \forall x\in(a,b)$

$\exists c : \frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$.

? $\frac{f'(\xi)}{\xi} = \frac{f(b)-f(a)}{\frac{(b+a)(b-a)}{2}} = \frac{f(b)-f(a)}{\frac{b^2-a^2}{2}} = \frac{f(b)-f(a)}{\frac{b^2}{2}-\frac{a^2}{2}}$

$\rightarrow g(x) = \frac{x^2}{2} \Rightarrow g'(x) = x$, $g'>0$ на (a,b)

$\overset{\text{Коши}}{\Longrightarrow} \exists\xi\in(a,b) \quad \frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(\xi)}{g'(\xi)} = \frac{f'(\xi)}{\xi}$.

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2) $a_n = \sqrt[n^{-3}]{1-\frac{1}{n}+\sin\frac{1}{n}}$

а) $\lim\limits_{n\to\infty} a_n = ?$

б) $\lim\limits_{n\to\infty} b_n = ?$, $b_n = \frac{a_n(\sqrt{1}+\sqrt{2}+\dots\sqrt{n})}{n^{\alpha+3/2}}$, $\alpha\in\mathbb{R}$

а)

$\sin\frac{1}{n} = \frac{1}{n} - \frac{1}{6n^3} + o\left(\frac{1}{n^4}\right)$, $n\to+\infty$

$a_n = \left(1-\frac{1}{n}+\sin\frac{1}{n}\right)^{1/n^3} = \left(1-\frac{1}{n}+\frac{1}{n}-\frac{1}{6n^3}+o\left(\frac{1}{n^4}\right)\right)^{1/n^3} =$

$= \left(1-\frac{1}{6n^3}+o\left(\frac{1}{n^4}\right)\right)^{1/n^3} = e^{1/n^3 \ln\left(1-\frac{1}{6n^3}+o\left(\frac{1}{n^4}\right)\right)} = e^{1/n^3\cdot\left(-\frac{1}{6n^3}+o\left(\frac{1}{n^4}\right)\right)}$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} e^{-\frac{1}{6n^6}+o\left(\frac{1}{n^7}\right)} = 1$

б) $\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} \frac{a_n(\sqrt{1}+\sqrt{2}+\dots+\sqrt{n})}{n^{\alpha+3/2}}$

$$\lim_{n\to\infty}\frac{\sqrt{1}+\sqrt{2}+\dots+\sqrt{n}}{n^{\alpha+3/2}} \overset{\alpha>-3/2}{\underset{\text{ШТ } *}{=}} \lim_{n\to+\infty}\frac{\sqrt{1}+\dots+\sqrt{n}+\sqrt{n+1}-(\sqrt{1}+\dots+\sqrt{n})}{(n+1)^{\alpha+3/2}-n^{\alpha+3/2}} =$$

$\alpha>-3/2 \implies n^{\alpha+3/2}\to+\infty,\ n\to+\infty$

$$= \lim_{n\to+\infty}\frac{\sqrt{n+1}}{(n+1)^{\alpha+3/2}-n^{\alpha+3/2}} = \lim_{n\to+\infty}\frac{n^{1/2}}{n^{\alpha+3/2}-(n-1)^{\alpha+3/2}} =$$

$$= \lim_{n\to+\infty}\frac{n^{1/2}}{n^{\alpha+3/2}\left(1-\left(1-\frac{1}{n}\right)^{\alpha+3/2}\right)} = \lim_{n\to\infty}\frac{n^{-\alpha-1}}{1-\left(1-\frac{1}{n}\right)^{\alpha+3/2}} =$$

$1-(\alpha+3/2)\cdot\frac{1}{n}+o\left(\frac{1}{n}\right)$

$$= \lim_{n\to\infty}\frac{1}{n^{\alpha+1}\left(\cancel{1}-\left(\cancel{1}-(\alpha+3/2)\cdot\frac{1}{n}+o\left(\frac{1}{n}\right)\right)\right)} = \lim_{n\to\infty}\frac{1}{n^{\alpha}\cdot\underset{\vee\, 0}{\left(\alpha+\frac{3}{2}\right)}+o(n^{\alpha})} = \begin{cases}0, & \alpha>0\\ \frac{2}{3}, & \alpha=0\\ +\infty, & \alpha<0\end{cases}$$

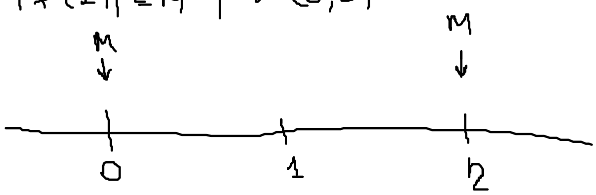
$\alpha\le-3/2$ $\quad \lim_{n\to\infty}\frac{\sqrt{1}+\dots+\sqrt{n}}{n^{\alpha+3/2}} = +\infty$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \underset{\downarrow\, 1}{a_n}\cdot\frac{\sqrt{1}+\dots+\sqrt{n}}{n^{\alpha+3/2}} = \begin{cases}+\infty, & \alpha<0\\ \frac{2}{3}, & \alpha=0\\ 0, & \alpha>0\end{cases}$$

(54) $f\in C[0,2]\cap D(0,2)$ $\quad \overset{?}{\implies} \quad f(x)\ge 0 \quad x\in[0,2]$

$f(0)=f(2)=M>0$

$|f'(x)|\le M$, $x\in(0,2)$



ППС. $\exists x\in(0,2)$ $\quad f(x)<0$

$f(2)-f(x) = M-f(x) > M$

$f(x)-f(0) = f(x)-M < -M$

$\left|\frac{f(2)-f(x)}{2-x}\right| > \frac{M}{2-x} \quad \xrightarrow{\text{Лагранж}} \quad \exists y_1\in(x,2) \quad |f'(y_1)| = \left|\frac{f(2)-f(x)}{2-x}\right| > \frac{M}{2-x}$

$\left|\frac{f(x)-f(0)}{x-0}\right| > \frac{M}{x} \quad \rightsquigarrow \quad \exists y_2\in(0,x) \quad |f'(y_2)| > \frac{M}{x}$

$\hookrightarrow |f'(y_1)| \leq M \Rightarrow \frac{M}{2-x} < M \Rightarrow 1 < 2-x$

$\underline{x < 1}$ $\Big\}$ овакво x не постоји ↯

$\hookrightarrow |f'(y_2)| \leq M \Rightarrow \frac{M}{x} < M \Rightarrow \underline{x > 1}$

$\Rightarrow f \geq 0.$