51. δ)

$$a_n = \frac{\left(\arcsin\frac{1}{n} - \sin\frac{1}{n} - \frac{1}{3n^3}\right)\cdot(n+3)^{3/2}}{\left((\cos\frac{1}{n})^{\sin\frac{1}{n^2}} - \left(1+\frac{1}{n^3}\right)^{\beta}\right)\sqrt{n}} \quad \sim \frac{1}{n^5}\cdot n^{3/2} = n^{-7/2}, \quad \sqrt{n} \sim n^{1/2}, \quad \beta \in \mathbb{R}$$

$$\lim_{n\to\infty} a_n = ?$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$$

$$x\to 0, \quad \arcsin x = \arcsin 0 + x + 0\cdot\frac{x^2}{2!} + \frac{x^3}{3!} + 0\cdot\frac{x^4}{4!} + 9\cdot\frac{x^5}{5!} + o(x^5)$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} = f'(x)$$

$$f''(x) = (1-x^2)^{-1/2} = -\frac{1}{2}\cdot(1-x^2)^{-3/2}\cdot(-2x) = x\cdot(1-x^2)^{-3/2}, \quad f''(0)=0$$

$$f'''(x) = (1-x^2)^{-3/2} + x\cdot\left(-\frac{3}{2}\right)(1-x^2)^{-5/2}\cdot(-2x) = (1-x^2)^{-3/2} + 3x^2(1-x^2)^{-5/2}, \quad f'''(0)=1$$

$$f^{(4)}(x) = 3x(1-x^2)^{-5/2} + 6x(1-x^2)^{-5/2} - \frac{15}{\cancel{2}}\cdot x^2(1-x^2)^{-7/2}(-\cancel{2}x) =$$

$$= \underbrace{9x(1-x^2)^{-5/2}} + 15x^3(1-x^2)^{-7/2} \qquad f^{(4)}(0)=0$$

$$f^{(5)}(x) = 9(1-x^2)^{-5/2} + 45x^2(1-x^2)^{-7/2} + 45x^2(1-x^2)^{-7/2} + 15x^3(\dots)$$

$$f^{(5)}(0) = 9$$

$$(n+3)^{3/2} = n^{3/2}\left(1+\frac{3}{n}\right)^{3/2} \sim n^{3/2}, \quad \frac{3}{n}\to 0$$

$$(1+t)^{\alpha} = 1 + \binom{\alpha}{1}t + \binom{\alpha}{2}t^2 + \dots, \quad t\to 0$$

$$\left(\cos\frac{1}{n}\right)^{\sin\frac{1}{n^2}} = e^{\sin\frac{1}{n^2}\ln\left(\cos\frac{1}{n}\right)}$$

$$a_n = \frac{\overbrace{\left(\arcsin\frac{1}{n} - \sin\frac{1}{n} - \frac{1}{3n^3}\right)}^{\sim\frac{1}{n^5}}\overbrace{(n+3)^{3/2}}^{\sim n^{3/2}}\overbrace{\cdot\frac{1}{\sqrt{n}}}^{\sim n^{-1/2}}}{\left(\left(\cos\frac{1}{n}\right)^{\sin\frac{1}{n^2}} - \left(1+\frac{1}{n^3}\right)^{\beta}\right)} \sim \quad \sim n^{-5+\frac{3}{2}-\frac{1}{2}} = n^{-4}$$

$$\sin t = t + o(t^2), \quad t\to 0$$

$$\sin\frac{1}{n^2} = \frac{1}{n^2} + o\left(\frac{1}{n^4}\right)$$

$$\cos\frac{1}{n} = 1 - \frac{t^2}{2} + \frac{t^4}{4!} + o(t^5), \quad t\to 0$$

$$\ln\left(\cos\frac{1}{n}\right) = \ln\Big(1 \underbrace{- \frac{1}{2n^2} + \frac{1}{4!n^4} + o\left(\frac{1}{n^5}\right)}_{\sim\frac{1}{n^2}}\Big) = \quad \ln(1+t) = t - \frac{t^2}{2} + o(t^2), \quad t\to 0$$

$$= -\frac{1}{2n^2} + \frac{1}{4!n^4} + o\left(\frac{1}{n^5}\right) - \frac{1}{2}\left(-\frac{1}{2n^2} + \frac{1}{4!n^4} + o\left(\frac{1}{n^5}\right)\right)^2 + o\left(\frac{1}{n^4}\right)$$

$$= -\frac{1}{2n^2} + \frac{1}{4!n^4} - \frac{1}{8n^4} + o\left(\frac{1}{n^4}\right) = -\frac{1}{2n^2} - \frac{1}{12n^4} + o\left(\frac{1}{n^4}\right)$$

$$4! = 24$$

$$e^t = 1 + t + o(t), \quad t\to 0$$

$$e^{\left(\frac{1}{n^2}+o\left(\frac{1}{n^4}\right)\right)\left(-\frac{1}{2n^2} - \frac{1}{12n^4} + o\left(\frac{1}{n^4}\right)\right)} = e^{-\frac{1}{2n^4} + o\left(\frac{1}{n^4}\right)} = 1 - \frac{1}{2n^4} + o\left(\frac{1}{n^4}\right)$$

$$\left(1+\frac{1}{n^3}\right)^{\beta} = 1+\underset{=\beta}{\binom{\beta}{1}}\cdot\frac{1}{n^3} + \underbrace{\binom{\beta}{2}\frac{1}{n^6}+o\left(\frac{1}{n^6}\right)}_{\text{није неопходно јер је } o\left(\frac{1}{n^4}\right)}$$

$$a_n = \frac{\frac{1}{n^4}+o\left(\frac{1}{n^4}\right)}{1-\frac{1}{2n^4}+o\left(\frac{1}{n^4}\right)-1-\beta\cdot\frac{1}{n^3}+o\left(\frac{1}{n^4}\right)} = \frac{\frac{1}{n^4}+o\left(\frac{1}{n^4}\right)>0}{-\frac{1}{2n^4}-\beta\frac{1}{n^3}+o\left(\frac{1}{n^4}\right)} = \begin{cases} \frac{o\left(\frac{1}{n^3}\right)}{-\beta\cdot\frac{1}{n^3}+o\left(\frac{1}{n^3}\right)}\cdot\frac{n^3}{n^3} & \beta\neq 0 \\ -2 & \beta=0 \end{cases}$$

$$= \begin{cases} \frac{o(1)\to 0^+}{-\beta+o(1)}, & \beta\neq 0 \\ -2, & \beta=0 \end{cases} = \begin{cases} +\infty & \beta<0 \\ -\infty & \beta>0 \\ -2 & \beta=0 \end{cases}$$

5Z_a)
$$f(x) = \begin{cases} \ln(1+3x^2)^{\,>0}, & x\le -1 \\ ax^2+bx+c, & -1<x\le 0 \\ \frac{\sin x}{\sqrt{x}}_{\,>0}, & x>0 \end{cases}$$

$\Rightarrow f$ добро деф

f непр? $\qquad f(-1) = \ln(1+3\cdot(-1)^2) = \ln 4$

$$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} ax^2+bx+c = +a-b+c = \ln 4$$

$f(0)=c$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\frac{\sin x}{\sqrt{x}} = \lim_{x\to 0^+}\sqrt{x}^{\,\to 0}\underbrace{\left(\frac{\sin x}{x}\right)}_{\to 1} = 0$$

$c=0$, $\quad a-b=\ln 4$

f диф? $\qquad f'(x) = \begin{cases} \frac{6x}{1+3x^2}, & x<-1 \\ 2ax+b, & -1<x<0 \\ \frac{\sqrt{x}\cos x - \sin x\cdot\frac{1}{2}\frac{1}{\sqrt{x}}}{x}, & x>0 \end{cases}$

$\Rightarrow f$ је диф на $(-\infty,-1)\cup(-1,0)\cup(0,+\infty)$

u $x=-1$:

$$f'_-(-1)=\lim_{h\to 0^-}\frac{f(x+h)-f(x)}{h} = \frac{6x}{1+3x^2}\Big|_{x=-1} = -\frac{6}{1+3} = -\frac{3}{2}$$

$$f'_+(-1) = \lim_{h\to 0^+}\frac{f(-1+h)-f(-1)}{h} = \lim_{h\to 0^+}\frac{a(-1+h)^2+b(-1+h)-\ln 4}{h} =$$

$$= \lim_{h\to 0^+}\frac{a(1^2+h^2-2h)-b+hb-\ln 4}{h} \underset{\substack{\downarrow \\ a-b=\ln 4}}{=} \lim_{h\to 0^+}\frac{ah^2-2h\cdot a+h\cdot b}{h} =$$

$$= b-2a$$

f gud u -1 $\Rightarrow$ $b-2a=-\frac{3}{2}$

$$a-b=\ln 4$$

$$b=\left(-\frac{3}{2}-\ln 4\right)\cdot\frac{1}{2} = -\frac{3}{4}-\ln 2$$

$$a = \underset{\substack{\shortparallel \\ 2\ln 2}}{\ln 4}-\frac{3}{4}-\ln 2 = \ln 2-\frac{3}{4}$$

$x=0$:

$$f'_-(0)=b$$

$$f'_+(0)= \lim_{h\to 0^+}\frac{f(h)-f(0)}{h} = \lim_{h\to 0^+}\frac{\frac{\sin h}{\sqrt{h}}-0}{h} = \lim_{h\to 0^+}\underset{\substack{\downarrow \\ 1}}{\frac{\sin h}{h}}\cdot\underset{\substack{\downarrow \\ +\infty}}{\frac{1}{\sqrt{h}}} = +\infty$$

$\Rightarrow$ f nije gud u 0.