51. в) $a_n = \underline{\underline{n^3}} \cdot \left(\sin\frac{1}{n} \ln\left(1 + \frac{1}{n} - \frac{1}{n^2}\right) - \underline{\underline{\frac{1}{n}}} \ln\left(1+\frac{1}{n}\right)\right)$

$\lim\limits_{n\to\infty} a_n = ?$

$$\sin\frac{1}{n} = \underline{\underline{\frac{1}{n}}} - \frac{1}{6n^3} + \theta\left(\frac{1}{n^4}\right), \; n\to\infty$$

$$\sin x = x - \frac{x^3}{6} + \theta(x^4), \; x\to 0$$

$$\theta\left(\frac{1}{n^2}\right)$$

$$\ln\Big(1 + \underbrace{\frac{1}{n} - \frac{1}{n^2}}_{x}\Big) = \left(\frac{1}{n} - \frac{1}{n^2}\right) - \frac{\left(\frac{1}{n} - \frac{1}{n^2}\right)^2}{2} + \theta\left(\left(\frac{1}{n} - \frac{1}{n^2}\right)^2\right) = *$$

$$\ln(1+x) = \underline{\underline{x}} - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n-1}\frac{x^n}{n} + \theta(x^n)$$

$$* = \frac{1}{n} - \frac{1}{n^2} - \frac{1}{2n^2} + \theta\left(\frac{1}{n^2}\right)$$

$$\ln\left(1+\frac{1}{n}\right) = \frac{1}{n} - \frac{1}{2n^2} + \theta\left(\frac{1}{n^2}\right)$$

$$\sin\frac{1}{n}\cdot\ln\left(1+\frac{1}{n} - \frac{1}{n^2}\right) - \frac{1}{n}\ln\left(1+\frac{1}{n}\right) =$$

$$= \left(\frac{1}{n} - \frac{1}{6n^3} + \theta\left(\frac{1}{n^4}\right)\right)\left(\frac{1}{n} - \frac{3}{2n^2} + \theta\left(\frac{1}{n^2}\right)\right) - \frac{1}{n}\left(\frac{1}{n} - \frac{1}{2n^2} + \theta\left(\frac{1}{n^2}\right)\right)$$

$$= \cancel{\frac{1}{n^2}} - \frac{3}{2n^3} + \theta\left(\frac{1}{n^3}\right) - \frac{1}{6n^4}\left(1 - \frac{3}{2n} + \theta\left(\frac{1}{n}\right)\right) + \theta\left(\frac{1}{n^4}\right)\left(\frac{1}{n} - \frac{3}{2n^2} + \theta\left(\frac{1}{n^2}\right)\right)$$

$$- \cancel{\frac{1}{n^2}} + \frac{1}{2n^3} + \theta\left(\frac{1}{n^3}\right)$$

$$\theta\left(\frac{1}{n^2}\right)\cdot\theta\left(\frac{1}{n^2}\right) = \theta\left(\frac{1}{n^4}\right) \text{ "}{=}\text{" } \theta\left(\frac{1}{n^2}\right)$$

$$\theta(f)\cdot\theta(g) = \theta(fg)$$

$$\alpha\cdot f\cdot\beta\cdot g = \alpha\cdot\beta\cdot f\cdot g$$

$$= -\frac{1}{n^3} + \theta\left(\frac{1}{n^3}\right)$$

$$\lim_{n\to\infty} n^3\cdot\Big(\underbrace{\sin\frac{1}{n}\ln\left(1+\frac{1}{n} - \frac{1}{n^2}\right) - \frac{1}{n}\ln\left(1+\frac{1}{n}\right)}_{-\frac{1}{n^3} + \theta\left(\frac{1}{n^3}\right)}\Big) = -1$$

$$\lim_{n\to\infty}\frac{1\cdot a_1 + \dots + n a_n}{n^2} \overset{\text{ШТ}}{=} \lim_{n\to\infty}\frac{(n+1)a_{n+1}}{(n+1)^2 - n^2} = \lim_{n\to\infty}\frac{(n+1)a_{n+1}}{2n+1} = -\frac{1}{2}.$$

51.

$$f(x)=\begin{cases} a\dfrac{\sqrt{\sin x^2-x^3}}{x^b}, & x<0\\ c, & x=0\\ x^{x^2}, & x>0\end{cases}$$

непрекидност фје

$$\lim_{x\to0^+} f(x)=\lim_{x\to0^+} x^{x^2}=\lim_{x\to0^+} e^{x^2\ln x}=1 \Rightarrow c=1$$

$$\lim_{x\to0^+} x^2\ln x=\lim_{x\to0^+}\frac{\ln x}{\frac{1}{x^2}} \overset{ЛП}{\underset{*}{=}} \lim_{x\to0^+}\frac{\frac{1}{x}}{\frac{1}{x^2}}=\lim_{x\to0^+} x=0$$

$$\lim_{x\to0^-} f(x)=a\lim_{x\to0^-}\frac{\sqrt{\sin x^2-x^3}}{x^b}=a\lim_{x\to0^-}\frac{\sqrt{x^2+o(x^4)-x^3}}{x^b}=a\lim_{x\to0^-}\frac{-x\sqrt{1-x+o(x^2)}}{x^b}=$$

$$\sin x^2=x^2+o(x^4),\ x\to0$$

$$=\begin{cases}-a, & b=1\\ 0, & b<1 \text{ или } a=0\\ \pm\infty \text{ или не постоји}, & b>1\end{cases}$$

ми хоћемо да $\lim_{x\to0^-} f(x)=1 \Rightarrow b=1,\ a=-1$

испитујемо диф.

$$f'_+(0)=\lim_{h\to0^+}\frac{f(h)-f(0)}{h}=\lim_{h\to0^+}\frac{e^{h^2\ln h}-1}{h}=\lim_{h\to0^+}\frac{1+h^2\ln h+o(h^2\ln h)-1}{h}=$$

$$=\lim_{h\to0^+} h\ln h+o(h\ln h)=0$$

$$\lim_{h\to0^+} h\ln h=\lim_{h\to0}\frac{\ln h}{\frac{1}{h}}\overset{ЛП}{\underset{*}{=}}\lim_{h\to0}\frac{\frac{1}{h}}{-\frac{1}{h^2}}=0$$

$$f'_-(0)=\lim_{h\to0^-}\frac{f(h)-f(0)}{h}=\lim_{h\to0^-}\frac{-\frac{\sqrt{\sin h^2+h^3}}{h}-1}{h}=\lim_{h\to0^-}\frac{-\sqrt{\sin h^2+h^3}-h}{h^2}=$$

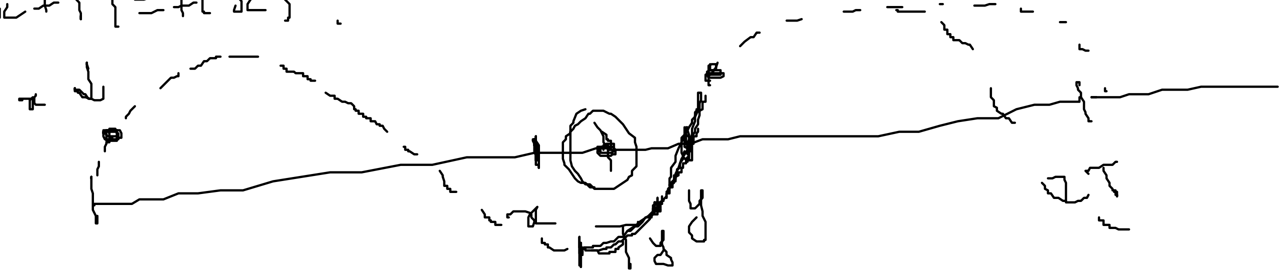
$$=\lim_{h\to0^-}\frac{-\sqrt{h^2+o(h^4)+h^3}-h}{h^2}=\lim_{h\to0^-}\frac{h\sqrt{1+h+o(h^2)}-h}{h^2}=$$

$$=\lim_{h\to0^-}\frac{h\cdot\left(1+\frac{1}{2}h+o(h^2)\right)-h}{h^2}=\frac{1}{2}\neq0=f'_+(0)$$

$\Rightarrow f$ није диф у 0.

(50.)

$f(x+T)=f(x)$



f непр на $\mathbb{R}$ $\Rightarrow$ f непр на $[0,2T]$

Кантор $\Rightarrow$ f равн непр на $[0,2T]$

$x, y \in [0,2T]$ $\quad |x-y|<\delta \Rightarrow |f(x)-f(y)|<\varepsilon$

$k \in \mathbb{Z}$

$x, y \in \mathbb{R}$ $\quad |x-y|<\delta \Rightarrow x-k\cdot 2T,\ y-k\cdot 2T \in [0,2T]$

$f(x)=f(x-k\cdot 2T)$

$f(y)=f(y-k\cdot 2T)$