

$$f(x) = x - \frac{2}{3}x + \left(\frac{2}{3}\right)^2 x - \left(\frac{2}{3}\right)^3 x + \dots + (-1)^n \left(\frac{2}{3}\right)^n x + (-1)^{n+1} f\left(\left(\frac{2}{3}\right)^{n+1} x\right)$$

ПММ ... за $\epsilon + \delta y$

$$f(x) = \lim_{n \rightarrow \infty} f(x) = \lim_{n \rightarrow \infty} x \left(1 - \frac{2}{3} + \left(\frac{2}{3}\right)^2 - \dots + (-1)^n \left(\frac{2}{3}\right)^n \right) + (-1)^{n+1} f\left(\left(\frac{2}{3}\right)^{n+1} x\right) = \frac{3}{5} x$$

$$\lim_{n \rightarrow \infty} \left(\frac{2}{3}\right)^{n+1} x = 0 \quad \left(\frac{2}{3} < 1\right)$$

$$f \text{ непрерывна} \Rightarrow \lim_{n \rightarrow \infty} f\left(\left(\frac{2}{3}\right)^{n+1} x\right) = f(0) = 0$$

$$A_n = 1 - \frac{2}{3} + \left(\frac{2}{3}\right)^2 - \dots + (-1)^n \left(\frac{2}{3}\right)^n = \frac{1 - q^{n+1}}{1 - q} = \frac{1 - \left(-\frac{2}{3}\right)^{n+1}}{5/3} \xrightarrow{n \rightarrow \infty} \frac{1}{5/3} = \frac{3}{5}$$

$q = -\frac{2}{3}$

49. 8) $f(x) = \sqrt{|x|} e^{-x}$ на $(0, +\infty)$ $\exists x, y$

$$|f(x) - f(y)| = |\sqrt{x} e^{-x} - \sqrt{y} e^{-y}| = |\sqrt{x} e^{-x} - \sqrt{y} e^{-x} + \sqrt{y} e^{-x} - \sqrt{y} e^{-y}|$$

$$\leq \underbrace{|\sqrt{x} - \sqrt{y}|}_{< 1} \cdot \underbrace{e^{-x}}_{\leq 1} + \sqrt{y} |e^{-x} - e^{-y}| = |\sqrt{x} - \sqrt{y}| \cdot e^{-x} + \sqrt{y} e^{-y} |e^{-x+y} - 1|$$

$\forall \epsilon > 0 \exists \delta > 0 \forall x, y \in (0, +\infty)$

$$\lim_{t \rightarrow 0} \frac{e^t - 1}{t} = 1 \quad \forall \epsilon > 0 \exists \delta > 0 \forall |t| < \delta \Rightarrow \left| \frac{e^t - 1}{t} - 1 \right| < \epsilon$$

$$|e^t - 1 - t| < \epsilon |t|$$

$$|e^t - 1| < |t| + \epsilon \cdot |t|$$

$$\epsilon = 1 \quad \exists \delta > 0 \quad \forall |t| < \delta \Rightarrow |e^t - 1| < 2|t|$$

$$|x - y| < \delta \Rightarrow |e^{-x+y} - 1| < 2|x - y|$$

$$\lim_{y \rightarrow +\infty} \sqrt{y} \cdot e^{-y} = \lim_{y \rightarrow +\infty} \frac{\sqrt{y}}{e^y} = 0 \Rightarrow \exists M > 0 \forall y > M \quad \frac{\sqrt{y}}{e^y} \leq 1$$

$$\Rightarrow \sqrt{y} e^{-y} |e^{-x+y} - 1| \leq 1 \cdot 2|x - y|, \quad y > M, \quad x > M, \quad |x - y| < \delta$$

$$e^{-x} \leq 1, \quad x \geq 0$$

$$|\sqrt{x} - \sqrt{y}| \leq \frac{1}{2}|x - y|, \quad x, y \geq 1$$

$$\Rightarrow |f(x) - f(y)| \leq \frac{1}{2}|x-y| + 2|x-y|, \quad |x-y| < \delta \text{ и } x, y > \max\{M, 1\}$$

$$|x-y| < \delta \Rightarrow |f(x) - f(y)| \leq \frac{3}{2}|x-y| < \varepsilon$$

$$\delta_1 = \min\left\{\delta, \frac{2}{3}\varepsilon\right\}$$

f равн непрерыв на $(M, +\infty)$

$\Rightarrow f$ равн непрерыв на \mathbb{R} .

Кантор f равн непрерыв на $[0, M+1]$

\downarrow имано непрерыв. промежуток где $f \neq y_0$.

$$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \sqrt{x} \cdot e^{-x} = 0 \Rightarrow f \text{ равн непрерыв на } [M, +\infty) \text{ за } M > 0.$$

Кантор f има непрерыв. ур. $y_0 = 0$ $\Rightarrow f$ равн непрерыв на $[0, M]$

$(0, +\infty)$

\downarrow
 f равн непрерыв на $(0, +\infty)$

$$x_n = \frac{a_n}{b_n}, \quad x_1 = 1$$

$$x_{n+1} = \frac{4x_n}{2x_n + 3}$$

$$\frac{a_{n+1}}{b_{n+1}} = \frac{4 \frac{a_n}{b_n}}{2 \frac{a_n}{b_n} + 3} = \frac{4a_n}{2a_n + 3b_n} \quad \left| \begin{array}{l} a_1 = 1 \\ b_1 = 1 \end{array} \right.$$

$$a_{n+1} = 4a_n, \quad a_1 = 1 \rightarrow a_2 = 4$$

$$a_3 = 4^2 = 16$$

$$a_4 = 4^3 \rightarrow a_n = 4^{n-1}, \quad n \in \mathbb{N}$$

$$b_{n+1} = 2a_n + 3b_n$$

\downarrow

$$2a_n = b_{n+1} - 3b_n$$

$$b_{n+2} - 3b_{n+1} = 4b_{n+1} - 12b_n \rightarrow b_n = \dots$$

$$b_n = a_n + \alpha a_{n-1} + \dots + \alpha^{n-1} a_1, \quad \alpha \in (0, 1)$$

$$\alpha = \frac{1}{\beta}, \quad \beta > 1$$

$$b_n = \frac{\alpha_1 y_n + \dots + \dots}{n}$$

$$b_n = 1 \cdot a_n + \frac{1}{\beta} a_{n-1} + \dots + \frac{1}{\beta^{n-1}} a_1 = \frac{\beta^{n-1} a_n + \dots + a_1}{\beta^n - \beta^{n-1}}$$

$$\lim_{n \rightarrow \infty} \frac{\beta^n a_{n+1}}{\beta^n - \beta^{n-1}} = \dots$$

$n\sqrt{2} \in [n\sqrt{2}] \subset n\sqrt{2}$

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36. $n \in \mathbb{C}$. $g_n \rightarrow +\infty$

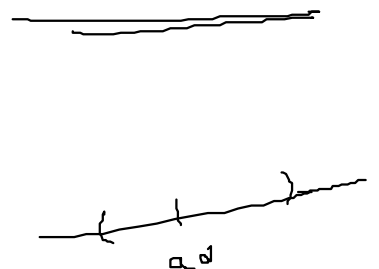
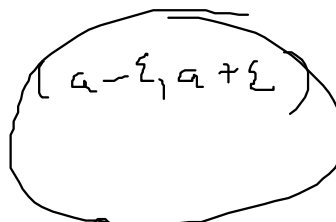
$$\left(g_n \rightarrow +\infty \quad \forall M > 0 \exists n_0 \in \mathbb{N} \forall n \geq n_0 \quad g_n > M \right)$$

$$\begin{aligned} & \exists M > 0 \forall n_0 \in \mathbb{N} \exists n \geq n_0 \quad g_n \leq M \\ & g_n \in \mathbb{N} \quad g_n \geq 1 \end{aligned}$$

$$g_n \in \mathbb{N} \quad g_{n_k} \in \{1, 2, \dots, M\} \Rightarrow \exists g_{n_k} : \lim_{k \rightarrow \infty} g_{n_k} = a$$

$$\frac{1}{g_n} \rightarrow \alpha$$

$$\varepsilon < 1$$



$$p_{n_k} \rightarrow \alpha$$

$$g_{n_k} = a \in \mathbb{N}$$

$$p_{n_k} \rightarrow \alpha \Rightarrow p_{n_k} \notin \mathbb{N}$$

$n \rightarrow \infty$ $x \rightarrow 0$

$$(x + \sigma(x))^n = \sum_{k=0}^n \binom{n}{k} x^k \cdot (\sigma(x))^{n-k}$$

$$= x^n + \binom{n}{n-1} x^{n-1} \cdot \sigma(x)$$

$$+ \binom{n}{n-2} x^{n-2} \sigma(x)^2$$

$$+ \dots + \sigma(x)^n = x^n + \sigma(x)^n$$

$$\sigma(x)^n = \sigma(x^n)$$

$$\sigma(x)^n = \sigma(x^n)$$

$$\sigma(x) \cdot \sigma(x) = \sigma(x^2)$$

$$\alpha \cdot \beta = \alpha \cdot \beta = x^2$$

$$\begin{aligned} \binom{n}{k} x^k \sigma(x)^{n-k} &= \binom{n}{k} x^k \sigma(x^{n-k}) \\ &= \binom{n}{k} \sigma(x^k \cdot x^{n-k}) = \binom{n}{k} \sigma(x^n) \\ &= \sigma(x^n) \end{aligned}$$