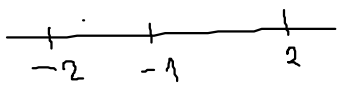
4. a) $\sqrt{\underbrace{x^2-4x+4}_{(x-2)^2}} + \sqrt{\underbrace{x^2+2x+1}_{(x+1)^2}} = \sqrt{\underbrace{x^2+4x+4}_{(x+2)^2}}$

$|x-2| + |x+1| = |x+2|$



1° $x < -2 \longrightarrow 2-x-1-x = -2-x$

$\longrightarrow 1-x = -2$

$x = 3$ ↯

2° $-2 \leq x < -1 \longrightarrow 2-x-x-1 = x+2$

$3x = -1 \longrightarrow x = -1/3$ ↯

3° $-1 \leq x < 2 \longrightarrow 2-x+x+1 = x+2$

$x = 1$ ✓

4° $x \geq 2 \longrightarrow x-2+x+1 = x+2 \Rightarrow x = 3$ ✓

b) $2^{x+2} > \left(\frac{1}{4}\right)^{1/x}$, $\frac{1}{4} = 2^{-2}$ $\rightsquigarrow 2^{x+2} > 2^{-2/x}$

$\Downarrow 2>1$

$x+2 > -2/x \rightsquigarrow x + 2/x + 2 > 0$

$x > 0 \Rightarrow x^2+2+2x > 0$

$(x+1)^2+1 > 0$ ✓

$x < 0 \Rightarrow x^2+2+2x < 0$ ↯

$\Rightarrow$ Решение je $x > 0$.

b) $\log_{1/3}(\log_4(x^2-5)) \geq 0 = \log_{1/3} 1$

$1/3 < 1 \Rightarrow \log_4(x^2-5) \leq 1 = \log_4 4$

$4 > 1 \Rightarrow x^2-5 \leq 4$

$x^2 \leq 9 \Rightarrow x \in [-3,3]$

$x^2-5 > 0 \Rightarrow x^2 > 5 \Rightarrow x \in (-\infty, -\sqrt{5}) \cup (\sqrt{5}, +\infty)$

$\log_4(x^2-5) > 0 \Rightarrow x^2-5 > 1 \Rightarrow x^2 > 6 \Rightarrow x \in [-\infty, -\sqrt{6}) \cup (\sqrt{6}, +\infty)$

$\Rightarrow$ Решение je $x \in [-3, -\sqrt{6}) \cup (\sqrt{6}, 3]$.

г) $\log_x \sqrt{x+12} > 1 = \log_x x$, $x > 0$, $x \neq 1$

1° $x \in (0,1) \Rightarrow \log_x \downarrow$ $\quad \sqrt{x+12} < x \uparrow^2$ $\quad x+12 < x^2$

$x^2 - x - 12 > 0$

$x_{1,2} = \frac{1 \pm \sqrt{1+48}}{2} = \frac{1 \pm 7}{2} \longrightarrow x_1 = -3$

$x_2 = 4$

$$x \in \left((-\infty, -3) \cup (4, +\infty)\right) \cap (0,1) = \emptyset$$

у овом случају нема решења

2° $x > 1 \Rightarrow \log_x \uparrow$ $\quad \sqrt{x+12} > x \longrightarrow x^2 - x - 12 < 0$

$$x \in (-3, 4) \cap (1, +\infty) = (1, 4)$$

Решење је $x \in (1, 4)$.

(5) Домен фје $f(x) = \operatorname{arctg}\left(\arcsin\left(\ln \frac{x+3}{x+1}\right)\right)$.

$\frac{x+3}{x+1} > 0$ и $-1 \le \ln \frac{x+3}{x+1} \le 1$ и $\arcsin\left(\ln \frac{x+3}{x+1}\right) \in \mathbb{R}$ ✓

$\Downarrow$

$x \in (-\infty, -3) \cup (-1, +\infty)$

$\underset{\ln \frac{1}{e}}{\underset{\shortparallel}{-1}} \le \ln \frac{x+3}{x+1} \quad / \ln \uparrow \quad \Rightarrow \quad \frac{1}{e} \le \frac{x+3}{x+1}$

$$0 \le \frac{x+3}{x+1} - \frac{1}{e} = \frac{ex + 3e - x - 1}{e(x+1)} = \frac{(e-1)\cdot x + (3e-1)}{e(x+1)}$$

$(e-1)x + (3e-1) = 0 \rightsquigarrow x = -\frac{3e-1}{e-1} < -1$

$$x \in \left(-\infty, -\frac{3e-1}{e-1}\right] \cup (-1, +\infty) \qquad -1 - \frac{2e}{e-1}$$

$\ln \frac{x+3}{x+1} \le \overset{\ln e}{1} \quad / \ln \uparrow \quad \Rightarrow \quad \frac{x+3}{x+1} \le e$

$$0 \le e - \frac{x+3}{x+1} = \frac{ex + e - x - 3}{x+1} = \frac{(e-1)x + (e-3)}{x+1}$$

$(e-1)x + (e-3) = 0 \rightarrow x = \frac{3-e}{e-1} > 0 > -1$

$$x \in (-\infty, -1) \cup \left[\frac{3-e}{e-1}, +\infty\right)$$

$$x \in \left((-\infty, -1) \cup \left[\frac{3-e}{e-1}, +\infty\right)\right) \cap \left((-\infty, -3) \cup (-1, +\infty)\right)$$

$$\cap \left(\left(-\infty, -\frac{3e-1}{e-1}\right] \cup (-1, +\infty)\right)$$

$$-\frac{3e-1}{e-1} \overset{?}{<} -3$$

$$\frac{-3e+1}{e-1} \overset{-3(e-1)-3}{=} -3 - \frac{3}{e-1} < -3$$

$$x \in \left(-\infty, -\frac{3e-1}{e-1}\right] \cup \left[\frac{3-e}{e-1}, +\infty\right).$$