19.9.2016. ?

$$\int_0^\infty \underbrace{\sin x \sin^2 x}_{\text{мења знак}} \, dx \quad \text{конв ?}$$

$$\int_0^\infty \sin^3 x\, dx = \lim_{\beta\to\infty} \int_0^\beta \sin^3 x\, dx = (*)$$

$$\int \sin^3 x\,dx = \int \sin^2 x \cdot \sin x\, dx = \int (1-\cos^2 x)\sin x\, dx = \begin{bmatrix} u = \cos x \\ du = \sin x\, dx \end{bmatrix}$$

$$= \int (1-u^2)\,(-du) = -u + \frac{u^3}{3} + C = -\cos x + \frac{\cos^3 x}{3} + C$$

$$= F(x)$$

$$(*) \quad \int_0^\beta \sin^3 x\, dx = \int_0^\pi + \int_\pi^{2\pi} + \dots + \int_{k\pi}^{\beta}, \quad k\pi \le \beta < (k+1)\pi$$

$$= F(x)\Big|_0^\pi + F(x)\Big|_\pi^{2\pi} + \dots + F(x)\Big|_{k\pi}^{\beta} = F(\beta) - F(0)$$

$$= \underbrace{-\cos\beta + \frac{\cos^3\beta}{3} - \left(-1+\frac{1}{3}\right)}_{\text{не постоји } \lim_{\beta\to+\infty}} \Rightarrow \int_0^{+\infty} \sin^3 x\, dx \text{ дивергира}$$

10.6.2019.

$$f \in C^2[0,1], \quad \int_0^1 f(x)\,dx = 3\int_{1/3}^{2/3} f(x)\,dx \overset{?}{\Rightarrow} \exists x_0 \in (0,1)\ f''(x_0) = 0$$

$$\int_0^1 f(x)\,dx - \int_{1/3}^{2/3} f(x)\,dx = 2\int_{1/3}^{2/3} f(x)\,dx$$

$$\int_0^{1/3} + \int_{2/3}^1 = 2\int_{1/3}^{2/3}$$

$$\underbrace{\int_{2/3}^1 f(x)\,dx}_{t = x - 1/3} - \int_{1/3}^{2/3} f(x)\,dx = \int_{1/3}^{2/3} f(x)\,dx - \underbrace{\int_0^{1/3} f(x)\,dx}_{t = x + \frac{1}{3}}$$

$$\int_{1/3}^{2/3} f(t+1/3) - f(t) \, dt = \int_{1/3}^{2/3} f(t) - f(t-1/3) \, dt$$

$$\int_{1/3}^{2/3} \underbrace{\left(f(t+1/3) - f(t) \right) - \left(f(t) - f(t-1/3) \right)}_{F \in C^2[1/3, 2/3]} dt = 0$$

$\Rightarrow F$ мења знак, $F \in C \Rightarrow \exists t_0 \quad F(t_0) = 0$

$$\underbrace{f(t_0+1/3) - f(t_0)}_{} = \underbrace{f(t_0) - f(t_0 - 1/3)}_{}$$

Лагранж f
$\exists \alpha \in (t_0, t_0 + 1/3)$ $\qquad$ $\exists \beta \in (t_0 - 1/3, t_0)$ $\quad \to \alpha \neq \beta$

Рол на f'

$$f'(\alpha) \cdot (\underbrace{t_0 + 1/3 - t_0}_{\cancel{1/3}}) = f'(\beta) \, \cancel{1/3} \quad \Rightarrow \quad f''(x_0) = 0$$

$x_0 \in (\alpha, \beta)$

октобар 2017

$f \in C^1[0,1] \qquad \int_0^1 f(x)\,dx = 0$

$\overset{?}{\Rightarrow} \forall \alpha \in (0,1) \quad \left| \int_0^\alpha f(x)\,dx \right| \leq \frac{1}{8} \max\limits_{0 \leq x \leq 1} |f'(x)|$

$$\int_0^\alpha f(x)\,dx = \begin{bmatrix} u = f & \to du = f'(x)\,dx \\ dv = dx & \to v = x \end{bmatrix} =$$

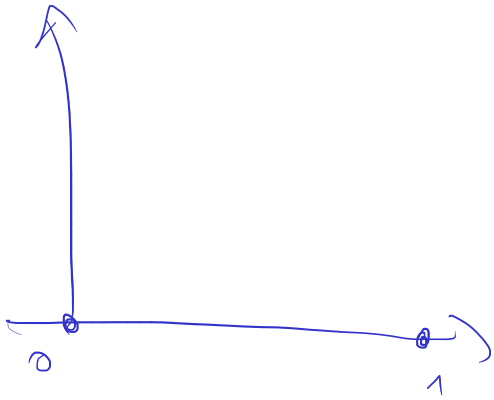
$$= x f(x) \Big|_0^\alpha - \int_0^\alpha x f'(x)\,dx = \alpha f(\alpha) - \int_0^\alpha x f'(x)\,dx$$

$$\int_0^1 f(x)\,dx = 0$$

$$\int_\alpha^1 f(x)\,dx = \int_0^1 - \int_0^\alpha = -\int_0^\alpha$$

$$F(\alpha) = \int_0^{\alpha} f(x)\,dx$$

$$\begin{cases} F \in C^2 \\ F(0) = 0, \quad F(1) = 0 = \int_0^1 f(x)\,dx \\ \Rightarrow |F(\alpha)| \le \underbrace{\frac{1}{8} \max|F''|}\ ? \end{cases}$$



$$F(\alpha) - F(0) = F'(\beta)\cdot(\alpha - 0)$$

$$F(1) - F(\alpha) = F'(\gamma)\,(1-\alpha)$$

$$|F(\alpha)| \le \underbrace{\min(\alpha, 1-\alpha)}_{\le \frac{1}{2}} \cdot \max|F'| \le \frac{1}{2} \max|F'|$$

$$|F'| \le \frac{1}{2} \max|F''|$$

$$\int_0^{\alpha} f(x)\,dx = \begin{bmatrix} u = f \\ dv = dx \end{bmatrix} = f(\alpha)\cdot\alpha - \int_0^{\alpha} x f'(x)\,dx$$

$$\int_0^{\alpha} x f'(x)\,dx = \begin{bmatrix} u = f' & \rightarrow du = f''\,dx \\ dv = x\,dx & \rightarrow v = \frac{x^2}{2} \end{bmatrix} =$$

$$= \frac{\alpha^2}{2} f'(\alpha) - \int_0^{\alpha} f''(x)\cdot\frac{x^2}{2}\,dx$$

??

ЈУН 2016.

$f:[0,1]\to\mathbb{R}$ непр, монотона

$?\forall a\in\mathbb{R} \quad \int_0^1 |f(x)-a|\,dx \geq \int_0^1 |f(x)-f(\tfrac{1}{2})|\,dx$?

WLOG $f\uparrow$

$1^\circ \quad a \geq f(1) \geq f(x)$

$$\int_0^1 a - f(x)\,dx \overset{?}{\geq} \int_0^{1/2} (f(\tfrac{1}{2}) - f(x))\,dx + \int_{1/2}^1 (f(x) - f(\tfrac{1}{2}))\,dx$$

$$a - \int_0^1 f(x)\,dx \overset{?}{\geq} \cancel{\tfrac{1}{2} f(\tfrac{1}{2})} - \int_0^{1/2} f(x)\,dx + \int_{1/2}^1 f(x)\,dx - \cancel{f(\tfrac{1}{2})\cdot\tfrac{1}{2}}$$

$$a \geq \int_0^1 - \int_0^{1/2} + \int_{1/2}^1 = 2\int_{1/2}^1 f(x)\,dx$$

$$\frac{a}{2} \overset{?}{\geq} \int_{1/2}^1 f(x)\,dx$$

$$f(x)\leq f(1) \Rightarrow \int_{1/2}^1 f(x)\,dx \leq \int_{1/2}^1 f(1)\,dx = \tfrac{1}{2} f(1) \leq \frac{a}{2} \quad \checkmark$$

$2^\circ \quad a \leq f(0)$ врв. слично као 1°

$3^\circ \quad f(0) < a < f(1) \Rightarrow \exists \alpha\in(0,1) \quad f(\alpha) = a$

$f\in C[0,1]$

$f\uparrow \Rightarrow \forall x\leq\alpha \quad f(x)\leq f(\alpha)=a$

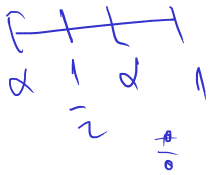
$x\geq\alpha \quad f(x)\geq f(\alpha)=a$

$$\int_0^1 |f(x)-a|\,dx = \int_0^1 |f(x)-f(\alpha)|\,dx =$$

$$= \int_0^\alpha f(\alpha)-f(x)\,dx + \int_\alpha^1 f(x)-f(\alpha)\,dx$$

$$= \alpha f(\alpha) - \int_0^\alpha f(x)\,dx + \int_\alpha^1 f(x)\,dx - (1-\alpha)f(\alpha) \overset{?}{\geq}$$

$$\frac{1}{2} f\left(\frac{1}{2}\right) - \frac{1}{2} f\left(\frac{1}{2}\right) + \int_{1/2}^1 f(x)\,dx - \int_0^{1/2} f(x)\,dx$$



$$\alpha f(\alpha) - (1-\alpha)f(\alpha) \overset{?}{\geq} \underbrace{\int_{1/2}^{\alpha} f(x)\,dx}_{\int_{1/2}^1 - \int_\alpha^1} - \underbrace{\int_\alpha^{1/2} f(x)\,dx}_{\int_0^{1/2} - \int_0^\alpha}$$

$$= 2\int_{1/2}^\alpha f(x)\,dx$$

3.1° $\alpha \geq \frac{1}{2}$ $\qquad \alpha f(\alpha) - (1-\alpha)f(\alpha) \geq 2\int_{1/2}^\alpha f(x)\,dx$

$$\int_{1/2}^\alpha f(x)\,dx$$

$$(2\alpha - 1)f(\alpha) \overset{?}{\geq} 2\int_{1/2}^\alpha f(x)\,dx$$

$$\left(\alpha - \frac{1}{2}\right)f(\alpha) \overset{?}{\geq} \int_{1/2}^\alpha f(x)\,dx \quad , \quad f\uparrow \quad f(x) \leq f(\alpha)$$

$$\int_{1/2}^\alpha f(x) \leq f(\alpha)\left(\alpha - \frac{1}{2}\right)$$

✓

II колок. 2017.

$f \in C^1(\mathbb{R})$

$\int_0^1 x^3 f(x)\,dx = 0$, f има лок. макс. у $\frac{1}{4}$

$\overset{?}{\Rightarrow} \exists \xi \quad f'(\xi) = 2f(1)$

f лок. макс. $+ f \in C^1$

$f'\left(\frac{1}{4}\right) = 0$

$$0 = \int_0^1 x^3 f(x)\,dx = \left[\begin{array}{l} u = f(x) \to du = f'(x)\,dx \\ dv = x^3\,dx \to v = \frac{x^4}{4} \end{array}\right]$$

$$= \underbrace{\frac{x^4}{4} \cdot f(x)\Big|_0^1} - \int_0^1 \frac{x^4}{4} \cdot f'(x)\,dx$$

$$= \frac{f(1)}{4} - \frac{1}{4}\int_0^1 x^4 \cdot f'(x)\,dx$$

$$f(1) = \int_0^1 x^4 f'(x)\,dx = f'(\xi) \underbrace{\int_0^1 x^4\,dx}_{\frac{1}{5}} =$$

Т о ср. вр.

$$\int_0^1 \underbrace{f(x)}_{\in C} \underbrace{g(x)}_{\substack{\geq 0 \\ \in C}}\,dx = f(\xi)\int_0^1 g(x)\,dx$$

$f'(\xi) = 5f(1)$

$f'\left(\frac{1}{4}\right) = 0$

Како је $2f(1)$ између $\underset{0}{\overset{f'(\frac{1}{4})}{\shortparallel}}$ и $5f(1) = f'(\xi)$

и $f' \in C(0,1) \Rightarrow \exists \zeta \in (\frac{1}{4}, \xi)$ $f'(\zeta) = 2f(1)$

Коши-Болцано

или Дарбу