$$\sum_{n=0}^{\infty} a_n (x-x_0)^n = f(x)$$

$f(x)$ сигурно конв у x_0, $f(x_0)=0$

$$T.R = \frac{1}{\overline{\lim\limits_{n\to\infty}} \sqrt[n]{|a_n|}} \overset{(*)}{=} \frac{1}{\lim\limits_{n\to\infty} \sqrt[n]{|a_n|}} \overset{(*)}{=} \lim_{n\to\infty} \left| \frac{a_n}{a_{n+1}} \right|$$

D - област конв реда $\quad (x_0-R, x_0+R) \subseteq D \subseteq [x_0-R, x_0+R]$

① $$\sum_{n=1}^{\infty} \frac{(x-1)^{2n}}{2^n n^3} = \sum_{n=1}^{\infty} a_n (x-1)^n \qquad a_{2n} = \frac{1}{2^n \cdot n^3}$$

$$a_{2n+1} = 0$$

$\sqrt[n]{a_n}$

$$\sqrt[2n]{a_{2n}} = \sqrt[2n]{\frac{1}{2^n \cdot n^3}} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt[2n]{n^3}} \to \frac{1}{\sqrt{2}}$$

$$\sqrt[n]{n} \to 1, \quad \left(\sqrt[n]{n}\right)^{3/2} \to 1$$

$$\sqrt[2n+1]{a_{2n+1}} = \sqrt[2n+1]{0} = 0$$

$$\overline{\lim_{n\to\infty}} \sqrt[n]{a_n} = \frac{1}{\sqrt{2}} \Rightarrow R = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

$x_0 = 1$

$$(1-\sqrt{2}, 1+\sqrt{2}) \subseteq D \subseteq [1-\sqrt{2}, 1+\sqrt{2}]$$

$x_1 = 1+\sqrt{2}$:

$x_1 \in D$?

$$\sum_{n=1}^{\infty} \frac{(1+\sqrt{2}-1)^{2n}}{2^n n^3} = \sum_{n=1}^{\infty} \frac{\sqrt{2}^{2n}}{2^n \cdot n^3} = \sum_{n=1}^{\infty} \frac{1}{n^3} \text{ конв.}$$

$\Rightarrow x_1 \in D$

$x_2 = 1-\sqrt{2}$

$$\sum_{n=1}^{\infty} \frac{(-\sqrt{2})^{2n}}{2^n n^3} = \sum_{n=1}^{\infty} \frac{1}{n^3} \text{ конв. } \quad p = 3 > 1$$

$x_2 \in D$

(2) $$\sum_{n=1}^{\infty} \underbrace{\frac{3^n \cdot n!}{n^n}}_{a_n} (x-e)^n$$

$$R = \frac{a_n}{a_{n+1}} = \frac{\frac{\cancel{3^n}\cdot \cancel{n!}}{n^n}}{\frac{3^{n+1}\cdot \cancel{(n+1)!}}{(n+1)^{n+1}}} = \frac{1}{3}\underbrace{\left(1+\frac{1}{n}\right)^n}_{\downarrow e} \to \frac{e}{3}$$

$$\left(\frac{2}{3}e, \frac{4}{3}e\right) \subseteq D \subseteq \left[\frac{2}{3}e, \frac{4}{3}e\right]$$

$$x_1 = \frac{4}{3}e : \sum_{n=1}^{\infty} \frac{3^n \cdot n!}{n^n}\Big(\underbrace{\frac{4}{3}e - e}_{e/3}\Big)^n = \sum_{n=1}^{\infty} \frac{\cancel{3^n}\cdot n!}{n^n}\cdot\frac{e^n}{\cancel{3^n}}$$

Стирлингова ф-ла $n! \sim \sqrt{2n\pi}\,\frac{n^n}{e^n}$

$$\frac{n!}{n^n}\cdot e^n \sim \sqrt{2n\pi}\cdot 1 \underset{n\to+\infty}{\not\to} 0$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{n!}{n^n} e^n \text{ дивергира} \Rightarrow \frac{4e}{3} \notin D$$

$$x_2 = \frac{2e}{3} : \sum_{n=1}^{\infty} (-1)^n \underbrace{\frac{n!\,e^n}{n^n}}_{\not\to 0} \Rightarrow \text{ дивергира } \frac{2e}{3} \notin D.$$

$$D = \left(\frac{2e}{3}, \frac{4e}{3}\right)$$

(3) $a > 0$, $\sum_{n=1}^{\infty} \underbrace{\frac{n!}{a^{n^2}}}_{a_n} x^n$, $x_0 = 0$

$$R = \frac{a_n}{a_{n+1}} = \frac{\frac{\cancel{n!}}{\cancel{a^{n^2}}}}{\frac{(n+1)\cancel{!}}{a^{(n+1)^2 = n^2+2n+1}}} = \frac{a^{2n+1}}{n+1} \xrightarrow[n\to+\infty]{} +\infty, \quad a > 1$$

$$\xrightarrow[n\to+\infty]{} 0 \quad 0 < a \le 1$$

$R = +\infty$

• $0 < a \leq 1 \qquad R = 0$

$$\underbrace{(x_0 - 0, x_0 + 0)}_{\emptyset} \subseteq D \subseteq \underbrace{[x_0 - 0, x_0 + 0]}_{\{x_0\}}$$

$$\boxed{D = \{x_0\} = \{0\}}$$

• $a > 1 \qquad R = +\infty$

$$(x_0 - \infty, x_0 + \infty) \subseteq D \subseteq [x_0 - \infty, x_0 + \infty]$$

$$\boxed{D = (-\infty, +\infty) = \mathbb{R}}$$

за вежбу: $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2} x^n, \ \sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n x^n, \ \sum_{n=1}^{\infty} \frac{(x-4)^n}{3^n (n+2)}$

$$\left[(1-x)^n = (-1)^n (x-1)^n\right]$$

4) $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{3^n (2n+1)(n+1)^p}$

$$a_n = \frac{(-1)^n}{3^n (2n+1)(n+1)^p}$$

$$R = \lim_{n \to \infty} \left| \frac{a_n}{a_{n+1}} \right| = \lim_{n \to \infty} \frac{\frac{1}{\cancel{3^n} (2n+1)(n+1)^p}}{\frac{1}{3^{\cancel{n}+1} (2n+3)(n+2)^p}} =$$

$$= \lim_{n \to \infty} \frac{3 \cdot (2n+3)(n+2)^p}{(2n+1)(n+1)^p} = 3$$

$$(-3, 3) \subseteq D \subseteq [-3, 3]$$

$x_1 = 3$: $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot \cancel{3^n}}{\cancel{3^n} (2n+1)(n+1)^p} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)(n+1)^p}$

$$\frac{1}{(2n+1)(n+1)^p} \sim \frac{1}{2n \cdot n^p} = \frac{1}{2n^{p+1}} \xrightarrow[n \to +\infty]{} 0 \iff p+1 > 0$$

$$p > -1$$

$\frac{1}{(2n+1)(n+1)^p} \downarrow$? $\quad (2n+1)(n+1)^p \uparrow$? $\quad n \geq n_0$

$f(x) = (2x+1)(x+1)^p \quad \uparrow \quad x \geq x_0$?

$f'(x) = 2(x+1)^p + p(x+1)^{p-1} \cdot (2x+1) = (x+1)^{p-1}(2x+2+2px+p)$

$= \underbrace{(x+1)^{p-1}}_{\frac{>0}{x>0}} \cdot \left(2\underbrace{(1+p)}_{>0} \cdot x + \underbrace{2+p}_{>0} \right) > 0 \quad p > -1, \; x > 0$

$f \uparrow$ за $x > 0$

$\Rightarrow (2n+1)(n+1)^p \uparrow \Rightarrow \frac{1}{(2n+1)(n+1)^p} \downarrow \quad p > -1$

$\overset{\text{Лајбниц}}{\Longrightarrow} \; p > -1 \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)(n+1)^p}$ конв

$3 \in D, \; p > -1$

$3 \notin D, \; p \leq -1$ јер општи члан реда $\not\to 0$.

$x_2 = -3$: $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot (-3)^n}{3^n \cdot (2n+1)(n+1)^p} = \sum_{n=1}^{\infty} \frac{1}{(2n+1)(n+1)^p}$

$\frac{1}{(2n+1)(n+1)^p} \sim \frac{1}{2n \cdot n^p} = \frac{1}{2n^{p+1}}, \quad \sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ конв. $\Leftrightarrow p+1 > 1$

II пор. крит. $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(2n+1)(n+1)^p}$ конв. $\Leftrightarrow p+1 > 1$, $p > 0$

$-3 \in D \Leftrightarrow p > 0$

$1^\circ \; p \leq -1 \qquad D = (-3, 3)$

$2^\circ \; -1 < p \leq 0 \qquad D = (-3, 3]$

$3^\circ \; p > 0 \qquad D = [-3, 3]$

5. $\sum\limits_{n=1}^{\infty} \underbrace{\left(\frac{a^n}{n}+\frac{b^n}{n^2}\right)}_{a_n} x^n \quad , \quad a,b>0$

1° $a \ge b$

$$\underbrace{\sqrt[n]{\frac{a^n}{n}}}_{=\,a\cdot\frac{1}{\sqrt[n]{n}}\,\to\, a} \le \sqrt[n]{a_n} = \sqrt[n]{\frac{a^n}{n}+\frac{b^n}{n^2}} \le \sqrt[n]{\frac{a^n}{n}+\frac{a^n}{n^2}}$$

$$\sqrt[n]{a_n} \xrightarrow{\text{ТоДП}} a$$

$$a\cdot\sqrt[n]{\frac{1}{n}+\underbrace{\frac{1}{n^2}}_{\le\frac{1}{n}}} \le a\cdot\sqrt[n]{\frac{2}{n}} \to a \qquad \frac{\sqrt[n]{2}\xrightarrow{n\to\infty}1}{\sqrt[n]{n}\to 1}$$

$$R = \frac{1}{\lim\limits_{n\to\infty}\sqrt[n]{a_n}} = \frac{1}{a}$$

$$\left(-\frac{1}{a},\frac{1}{a}\right) \subseteq D \subseteq \left[-\frac{1}{a},\frac{1}{a}\right]$$

$x_1=\frac{1}{a}$: $\sum\limits_{n=1}^{\infty}\left(\frac{a^n}{n}+\frac{b^n}{n^2}\right)\frac{1}{a^n} = \sum\limits_{n=1}^{\infty}\left(\frac{1}{n}+\frac{\left(\frac{b}{a}\right)^n}{n^2}\right) \overset{(*)}{=} \underbrace{\sum\limits_{n=1}^{\infty}\frac{1}{n}}_{\text{див.}} + \underbrace{\sum\limits_{n=1}^{\infty}\frac{\left(\frac{b}{a}\right)^n}{n^2}}_{\text{конв.}}$

по I поредб. крит.

$$\frac{\left(\frac{b}{a}\right)^n}{n^2} \le \frac{1}{n^2} \qquad \frac{b}{a}\le 1$$

$\Rightarrow \sum\limits_{n=1}^{\infty}\left(\frac{a^n}{n}+\frac{b^n}{n^2}\right)\frac{1}{a^n}$ дивергира

$\frac{1}{a}\notin D$

$x_2=-\frac{1}{a}$: $\sum\limits_{n=1}^{\infty}\left(\frac{a^n}{n}+\frac{b^n}{n^2}\right)\left(-\frac{1}{a}\right)^n = \sum\limits_{n=1}^{\infty}\left(\frac{(-1)^n}{n}+\frac{(-1)^n\left(\frac{b}{a}\right)^n}{n^2}\right) =$

$$= \underbrace{\sum\limits_{n=1}^{\infty}\frac{(-1)^n}{n}}_{\text{конв.}} + \underbrace{\sum\limits_{n=1}^{\infty}\frac{(-1)^n\left(\frac{b}{a}\right)^n}{n^2}}_{\text{конв. апс.}}$$

по Лајбницу

$\Rightarrow \sum\limits_{n=1}^{\infty}\left(\frac{a^n}{n}+\frac{b^n}{n^2}\right)\left(-\frac{1}{a}\right)^n$ конв $\Rightarrow -\frac{1}{a}\in D$

$D=\left[-\frac{1}{a},\frac{1}{a}\right)$.

2° $b > a$

$$b\cdot\frac{1}{(\sqrt[n]{n})^2} = \sqrt[n]{\frac{b^n}{n^2}} \le \sqrt[n]{a_n} = \sqrt[n]{\frac{a^n}{n}+\frac{b^n}{n^2}} \underset{a<b}{\le} b\cdot\underbrace{\sqrt[n]{\frac{1}{n}+\frac{1}{n^2}}}_{\downarrow\, 1}$$

$$\downarrow \qquad\quad \downarrow \text{ Т. о 2П.} \qquad\quad \downarrow$$

$$b \qquad\quad b \qquad\quad b$$

$$R = \frac{1}{b}, \qquad \left(-\frac{1}{b}, \frac{1}{b}\right) \subseteq D \subseteq \left[-\frac{1}{b}, \frac{1}{b}\right]$$

$$x_1 = \frac{1}{b} \qquad \sum_{n=1}^{\infty}\left(\frac{a^n}{n}+\frac{b^n}{n^2}\right)\frac{1}{b^n} = \sum_{n=1}^{\infty}\left(\frac{\left(\frac{a}{b}\right)^n}{n}+\frac{1}{n^2}\right) =$$

$$= \underbrace{\sum_{n=1}^{\infty}\frac{\left(\frac{a}{b}\right)^n}{n}} + \underbrace{\sum_{n=1}^{\infty}\frac{1}{n^2}}_{\text{конв.}} \Rightarrow \frac{1}{b} \in D$$

$$q = \frac{a}{b} < 1, \qquad \frac{\left(\frac{a}{b}\right)^n}{n} \le \left(\frac{a}{b}\right)^n$$

$$\sum_{n=1}^{\infty} q^n \text{ конв.} \overset{\text{I п.к.}}{\Longrightarrow} \sum_{n=1}^{\infty}\frac{\left(\frac{a}{b}\right)^n}{n} \text{ конв.}$$

$$x_2 = -\frac{1}{b} \qquad \sum_{n=1}^{\infty}(-1)^n\left(\frac{\left(\frac{a}{b}\right)^n}{n}+\frac{1}{n^2}\right) \rightarrow \text{апс. конв.} \Rightarrow -\frac{1}{b} \in D$$

$$D = \left[-\frac{1}{b}, \frac{1}{b}\right]$$

Тејлоров развој фје $f \in C^{\infty}(D_f)$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$$ — Тејлоров развој фје f у x_0

$$\overset{\shortparallel}{F(x)}$$

$f^{(0)}(x_0) := f(x_0)$

$0! := 1$

$x_0 = 0$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$ — Маклоренов развој

$D_f \to$ домен фје f

1° $F(x)$ дивергира у x_1, $x_1 \in D_f$

2° $F(x)$ конв. у $x_1 \in D_f$, $f(x_1) \neq F(x_1)$

3° $F(x)$ конв у $x_1 \in D_f$, $\underline{\underline{f(x_1) = F(x_1)}}$

Ако важи 3° за све $x_1 \in A \subseteq D_f$, онда кажемо да је f аналит. фја на A (ту је можемо посматрати као степени ред)

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad x \in \mathbb{R}$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad x \in \mathbb{R}$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \quad x \in \mathbb{R}$$

$$\ln(1+x) = \sum_{n=1}^{+\infty} \frac{(-1)^{n-1} x^n}{n}, \quad x \in (-1,1]$$

$$(1+x)^\alpha = \sum_{n=0}^{+\infty} \binom{\alpha}{n} x^n, \quad x \in \underline{\underline{D_\alpha}}$$

$\to$ испитаћемо шта је ово у зависности од α

- $\alpha \in \mathbb{N} \cup \{0\}$: $D_f = \mathbb{R}$
- $\alpha \notin \mathbb{N} \cup \{0\} \longrightarrow R = 1$
- $\alpha > 0,\ \alpha \notin \mathbb{N}$: $D_f = [-1, 1]$
- $-1 < \alpha < 0$: $D_f = (-1, 1]$
- $\alpha \leq -1$: $D_f = (-1, 1)$

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(1) Наћи Тејлоров развој f у тачки x_0

a) $f(x) = \frac{x+1}{2x+3}$, $x_0 = -2$ → за вежбу

б) $f(x) = \ln(3x-5)$, $x_0 = 3$

$$f(x) = \ln(3x-5),\quad x_0 = 3$$

$$\sum \underline{\quad} (\underbrace{x - x_0}_{t})^n$$

$$\sum \underline{\quad} x^n$$

$$t = x - x_0 = x - 3,\quad x = t + 3$$

$$g(t) = f(x)$$

$$g(t) = f(t+3) = \ln(3t + 9 - 5) = \ln(3t+4)$$ → Маклоренов развој ?

$$= \ln\left(4\left(\tfrac{3}{4}t + 1\right)\right) = \ln 4 + \underbrace{\ln\left(\tfrac{3}{4}t + 1\right)}$$

$$u = \frac{3}{4}t$$

$$\ln(1+u) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} u^n}{n},\quad u \in (-1, 1]$$

$$\ln\left(1 + \tfrac{3}{4}t\right) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} 3^n t^n}{4^n n},\quad \tfrac{3}{4}t \in (-1, 1]$$

$$t \in \left(-\tfrac{4}{3}, \tfrac{4}{3}\right]$$

$$g(t) = \ln 4 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1} 3^n t^n}{4^n n}, \quad t \in \left(-\tfrac{4}{3}, \tfrac{4}{3}\right]$$

$$f(x) = g(\underbrace{x-3}_{t}) = \ln 4 + \sum_{n=1}^{\infty} \underbrace{\frac{(-1)^{n-1} 3^n}{4^n \cdot n}}_{f^{(n)}(3)} (x-3)^n, \quad x-3 \in \left(-\tfrac{4}{3}, \tfrac{4}{3}\right]$$

$$x \in \left(\tfrac{5}{3}, \tfrac{13}{3}\right]$$

Тејлоров развој фје f у 3

Нaћи

(2) Маклоренов развој фје f:

а) $f(x) = e^{3x^2+1}$ → за вежбу

б) $f(x) = \dfrac{x}{3x+2}$

в) $f(x) = \sin x \cos x = \frac{1}{2}\sin 2x$

г) $f(x) = \operatorname{sh} x = \dfrac{e^x - e^{-x}}{2}$ } за вежбу

д) $f(x) = \operatorname{ch} x = \dfrac{e^x + e^{-x}}{2}$

в) $f(x) = \frac{1}{2}\sin 2x$

$$\sin t = \sum_{n=0}^{+\infty} \frac{(-1)^n t^{2n+1}}{(2n+1)!}, \quad t \in \mathbb{R}$$

$t = 2x$

$$f(x) = \frac{1}{2}\sum_{n=0}^{+\infty} \frac{(-1)^n (2x)^{2n+1}}{(2n+1)!}, \quad x \in \mathbb{R}$$

$$= \sum_{n=0}^{+\infty} \frac{(-1)^n \cdot 2^{2n} \cdot x^{2n+1}}{(2n+1)!}, \quad x \in \mathbb{R}.$$

б) $$f(x) = \frac{x}{3x+2} = x \cdot \frac{1}{3x+2} = x(3x+2)^{-1} = x \cdot \frac{1}{2}(1 + \underbrace{\tfrac{3}{2}x}_{t})^{-1}$$

$$(1+t)^{-1} = \sum_{n=0}^{+\infty} \binom{-1}{n} t^n, \quad t \in (-1,1)$$

$$\binom{-1}{n} = \frac{-1 \cdot (-1-1) \cdots (-1-n+1)}{n!} = \frac{(-1)^n \cdot n!}{n!} = (-1)^n$$

$$f(x)=\frac{1}{2}x\cdot\sum_{n=0}^{+\infty}(-1)^n\left(\frac{3}{2}x\right)^n=\sum_{n=0}^{+\infty}\frac{(-1)^n\cdot 3^n}{2^{n+1}}\cdot x^{n+1}$$

$$=\sum_{n=1}^{+\infty}\frac{(-1)^{n-1}3^{n-1}}{2^n}x^n \quad \left|\ \frac{3}{2}x\in(-1,1)\right.$$

$$x\in\left(-\frac{2}{3},\frac{2}{3}\right).$$

③ Израчунати:

a) $$A=\underbrace{\sum_{n=1}^{\infty}\frac{n+1-1}{3^n}}_{\text{конвергира}}=\sum_{n=1}^{\infty}\frac{n+1}{3^n}-\underbrace{\sum_{n=1}^{\infty}\frac{1}{3^n}}_{\text{конв.}}=3A-1-\underbrace{\sum_{n=1}^{\infty}\frac{1}{3^n}}$$

$$\frac{1}{3}\sum_{n=0}^{+\infty}\frac{1}{3^n}=\frac{1}{3}\,\frac{1}{1-\frac{1}{3}}=\frac{1}{2}$$

$$\frac{n+1}{3^{n+1}}$$

$$\sum_{n=1}^{\infty}\frac{n+1}{3^n}=3\sum_{n=1}^{\infty}\frac{n+1}{3^{n+1}}=3\left(\underbrace{\sum_{n=1}^{+\infty}\frac{n}{3^n}}_{A}-\frac{1}{3}\right)=3A-1$$

$$A=3A-\frac{3}{2}\implies 2A=\frac{3}{2}$$

$$A=3/4$$

б) $$\sum_{n=1}^{\infty}\frac{n^2}{3^n}\to \text{за вежбу}$$

в) $$\sum_{n=1}^{\infty}\frac{\overset{=x^n}{2^n}(n+1)}{n!}=\sum_{n=1}^{\infty}\frac{\overset{2^{n-1}}{2^n}\cancel{n}}{\underset{(n-1)!}{\cancel{n!}}}+\sum_{n=1}^{\infty}\underbrace{\frac{2^n}{n!}}_{a_n}=2\sum_{n=1}^{\infty}\frac{2^{n-1}}{(n-1)!}+\sum_{n=1}^{\infty}\frac{2^n}{n!}$$

$$\Gamma\ \frac{a_n}{a_{n+1}}=\frac{\frac{2^{\cancel{n}}}{\cancel{n!}}}{\frac{2^{n+1}}{(n+1)!}}=\frac{n+1}{2}\to+\infty$$

$$\frac{a_{n+1}}{a_n}\to 0\ \ \rfloor$$

$$=2\underbrace{\sum_{n=0}^{\infty}\frac{2^n}{n!}}_{2e^2}+\underbrace{\sum_{n=1}^{\infty}\frac{2^n}{n!}}_{e^2-1}$$

$$=3e^2-1$$

$$\left(\underbrace{\sum_{n=0}^{+\infty}\frac{x^n}{n!}}_{e^x}\right)\Bigg|_{x=2}=e^2$$

ž) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n(n+1)(n+2)}$

g) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$

$$\frac{1}{n(n+1)(n+2)} = \frac{A}{n} + \frac{B}{n+1} + \frac{C}{n+2} \quad , A, B, C = ?$$

$$A = C = \frac{1}{2} \quad , B = -1$$

$$= \frac{1}{2n} - \frac{1}{n+1} + \frac{1}{2(n+2)}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n(n+1)(n+2)} = \sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{2n} - \frac{1}{n+1} + \frac{1}{2(n+2)} \right) =$$

$\ln(1+x)$, $x = 1$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n}{2n} - \underbrace{\sum_{n=1}^{\infty} \frac{(-1)^n}{n+1}}_{\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{n}} + \underbrace{\sum_{n=1}^{\infty} \frac{(-1)^n}{2(n+2)}}_{-\frac{1}{2}\sum_{n=3}^{\infty} \frac{(-1)^{n-1}}{n}} \longrightarrow$$

сва 3 реда
з десна конв.
по лајбницу

$$\frac{1}{2} \underbrace{\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cdot 1^n}{n}}_{\ln(1+1)} \qquad \underbrace{\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{n}}_{\ln(1+1) - \frac{1}{1}} \qquad -\frac{1}{2} \underbrace{\sum_{n=3}^{\infty} \frac{(-1)^{n-1}}{n}}_{\ln(1+1) - \left(1 - \frac{1}{2}\right)}$$

$$= \ldots$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)} = \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n} - \sum_{n=1}^{\infty} \frac{1}{n+1} + \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n+2}$$

овде не може јер редови дивергирају

онда идемо на lim

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k(k+1)(k+2)} = \lim_{n\to\infty} \left(\frac{1}{2}\sum_{k=1}^{n} \frac{1}{k} - \sum_{k=1}^{n} \frac{1}{k+1} + \frac{1}{2}\sum_{k=1}^{n} \frac{1}{k+2} \right)$$

$$\ldots = \lim_{n\to\infty} \left(\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} - \frac{1}{2} - \underbrace{\frac{1}{n+1} + \frac{1}{2(n+1)} + \frac{1}{2(n+2)}}_{\to 0} \right) = \frac{1}{4}.$$