8 - понедељак 11.04.2022.

(3) $A = \begin{bmatrix} -1 & 3 \\ 5 & -3 \end{bmatrix}$

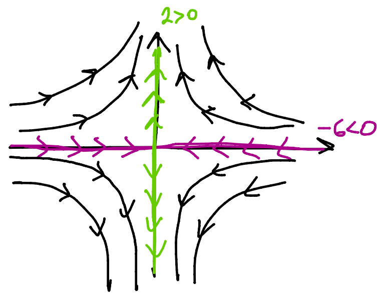
$-x_1+3x_2 = 5x_1-3x_2 = 0 \Rightarrow x_1 = x_2 = 0$, $X^* = (0,0)$

$\lambda_1 = -6$

$\lambda_2 = 2$ (реалне и разл. знака)

седло

$J = \begin{bmatrix} -6 & \\ & 2 \end{bmatrix}$



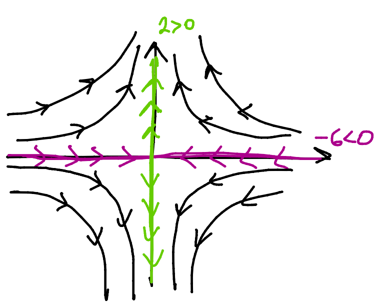
$\lambda_1 = -6 \rightsquigarrow \vartheta_1 = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$

$\lambda_2 = 2 \rightsquigarrow \vartheta_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

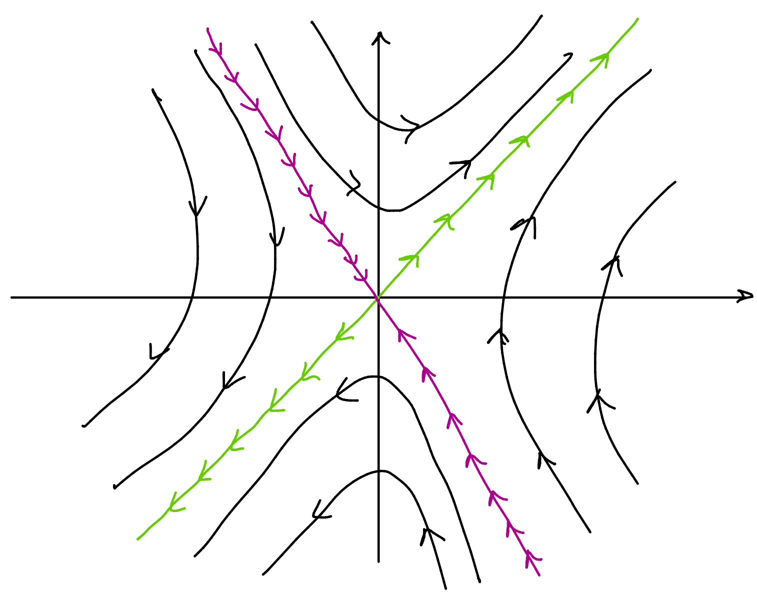
$X(t) = c_1 e^{-6t} \begin{bmatrix} 3 \\ -5 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $c_1, c_2 \in \mathbb{R}$

$c_1 = 0:$ $c_2 = 1:$ $X(t) = \begin{bmatrix} e^{2t} \\ e^{2t} \end{bmatrix}$ $x_1 = x_2 > 0$

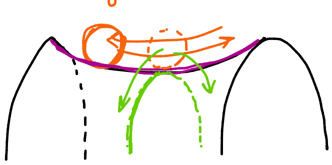
$c_1 = 1:$ $c_2 = 0:$ $X(t) = \begin{bmatrix} 3e^{-6t} \\ -5e^{-6t} \end{bmatrix}$ $5x_1 + 3x_2 = 0$, $x_1 > 0$



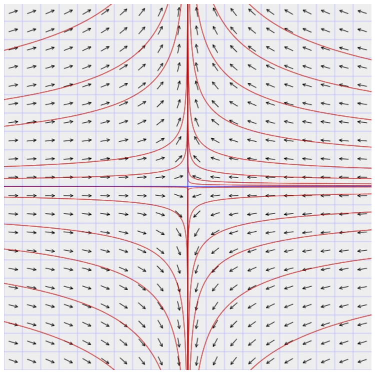
$\rightsquigarrow$

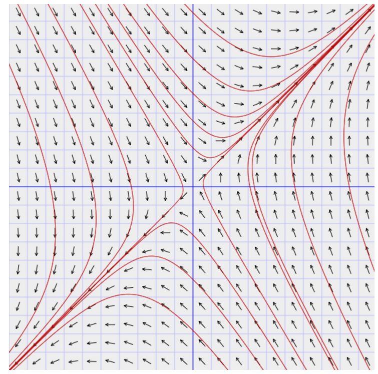


седло:



J ↷ A ↷





2. Скицирати фазни портрет динамичког система $X' = AX$, ако је:

(1) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$; (2) $A = \begin{bmatrix} -2 & -5 \\ 2 & 2 \end{bmatrix}$;

(3) $A = \begin{bmatrix} -1 & -1 \\ 2 & -3 \end{bmatrix}$. (4) $A = \begin{bmatrix} 7 & -10 \\ 4 & -5 \end{bmatrix}$.

(1) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$\hookrightarrow \lambda_{1/2} = \pm i$

$Re(\lambda_{1/2}) = 0$ – центар $\leftarrow X^* = (0,0)$

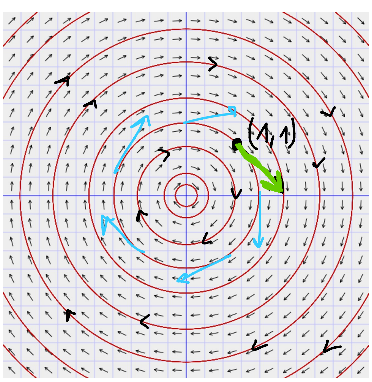
$x_1(t)$

$$X(t) = c_1 \begin{bmatrix} \cos t \\ -\sin t \end{bmatrix} + c_2 \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}, \quad c_1, c_2 \in \mathbb{R}$$

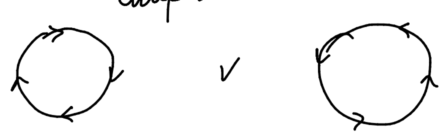
$x_2(t)$

испоставља се: $x_1(t)^2 + x_2(t)^2 = const = c_1^2 + c_2^2$

↳ кругови → периодично



Смер?



(1,1) ⇝ како изгледа извод? $X' = AX$

$$X = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow X' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

(2) $A = \begin{bmatrix} -2 & -5 \\ 2 & 2 \end{bmatrix} \rightsquigarrow \lambda_{1/2} = \pm i\sqrt{6}$

$Re(\lambda_{1/2}) = 0 \to$ центар $X^* = (0,0)$

(2) $A = \begin{bmatrix} & \\ 2 & 2 \end{bmatrix} \rightsquigarrow \lambda_{1/2} = \pm i\sqrt{6}$

$\mathrm{Re}(\lambda_{1/2}) = 0 \to$ центар $\qquad X^* = (0,0)$

$(A - i\sqrt{6}E)\gamma = 0$

$\therefore \gamma = \begin{bmatrix} 5 \\ -2 - i\sqrt{6} \end{bmatrix}$

$\psi(t) = e^{i\sqrt{6}t} \cdot \gamma = \left(\cos(\sqrt{6}t) + i\sin(\sqrt{6}t)\right) \cdot \begin{bmatrix} 5 \\ -2 - i\sqrt{6} \end{bmatrix} \rightsquigarrow \mathrm{Re}\,\psi, \ \rightsquigarrow \mathrm{Im}\,\psi$

$$x(t) = c_1 \cdot \begin{bmatrix} 5\cos(\sqrt{6}t) \\ -2\cos(\sqrt{6}t) + \sqrt{6}\sin(\sqrt{6}t) \end{bmatrix} + c_2 \cdot \begin{bmatrix} 5\sin(\sqrt{6}t) \\ -2\sin(\sqrt{6}t) - \sqrt{6}\cos(\sqrt{6}t) \end{bmatrix}, \ c_1, c_2 \in \mathbb{R}$$

↳ тешко нацртати

$$A = T \cdot \begin{bmatrix} 0 & \sqrt{6} \\ -\sqrt{6} & 0 \end{bmatrix} \cdot T^{-1}$$

$X' = AX = TJT^{-1}X \to \underbrace{T^{-1}X'}_{y'} = J\underbrace{T^{-1}X}_{y}$

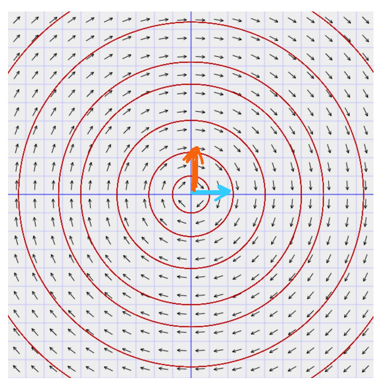
$y = T^{-1}X$

$y' = T^{-1}X'$

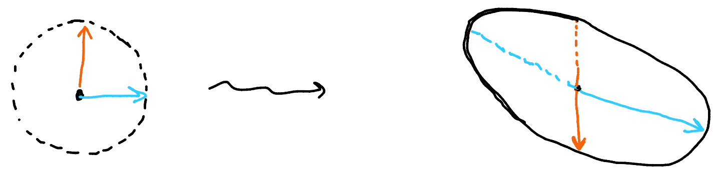
$y' = Jy$

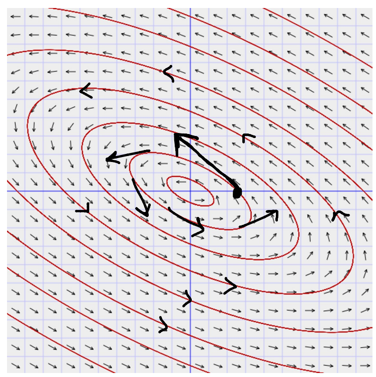
↳ знамо да су кругови

$J = \begin{bmatrix} 0 & \sqrt{6} \\ -\sqrt{6} & 0 \end{bmatrix} \to$ ово нацртамо



$T = [\mathrm{Re}\,\gamma \ \ \mathrm{Im}\,\gamma] = \begin{bmatrix} 5 & 0 \\ -2 & -\sqrt{6} \end{bmatrix}$





смер:

$$X=\begin{bmatrix}1\\0\end{bmatrix}$$

$$X'=AX=\begin{bmatrix}-2 & -5\\ 2 & 2\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}-2\\2\end{bmatrix}$$

(3) $A=\begin{bmatrix}-1 & -1\\ 2 & -3\end{bmatrix}$

$\lambda_{1/2}=-2\pm i$

$X^*=(0,0)$

$Re(\lambda_{1/2})\neq 0$ спирала (фокус)

$Re(\lambda_{1/2})>0$: нестабилна спирала = фт иду од X^*

$Re(\lambda_{1/2})<0$: стабилна спирала = фт иду у X^*

код нас

$A=TJT^{-1}$

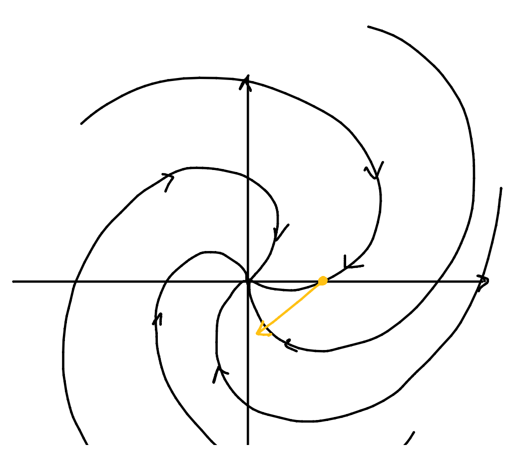
$$J=\begin{bmatrix}-2 & 1\\ -1 & -2\end{bmatrix}$$

$$T=\begin{bmatrix}1 & 0\\ 1 & -1\end{bmatrix}$$

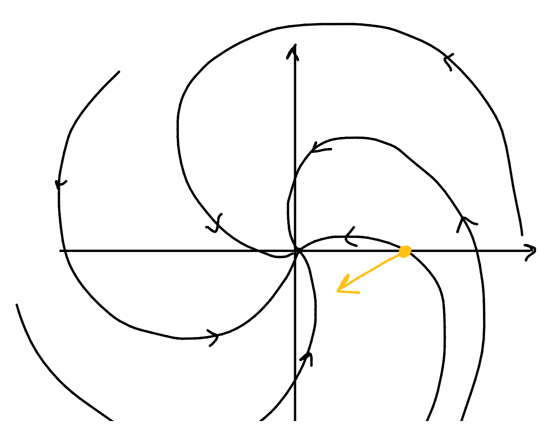
$(A-(-2+i)E)v=0$

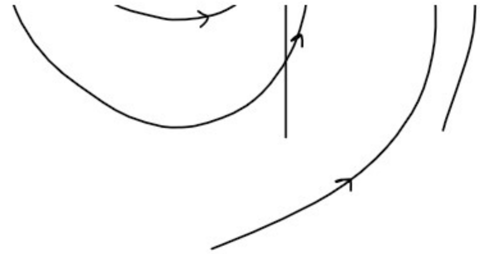
$$v=\begin{bmatrix}1\\1-i\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}+i\begin{bmatrix}0\\-1\end{bmatrix}$$

$J \rightsquigarrow$



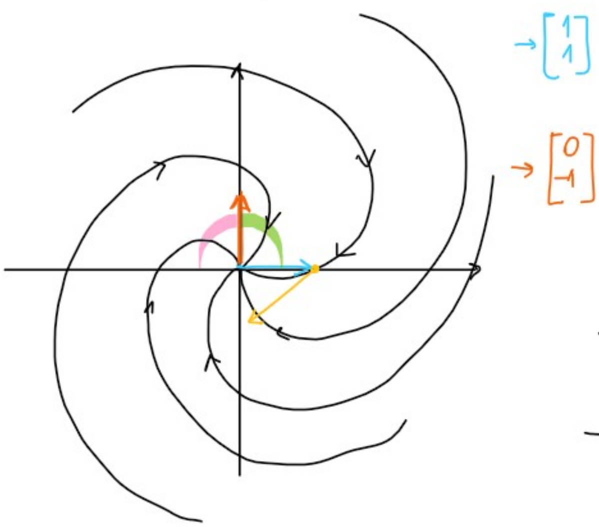
V



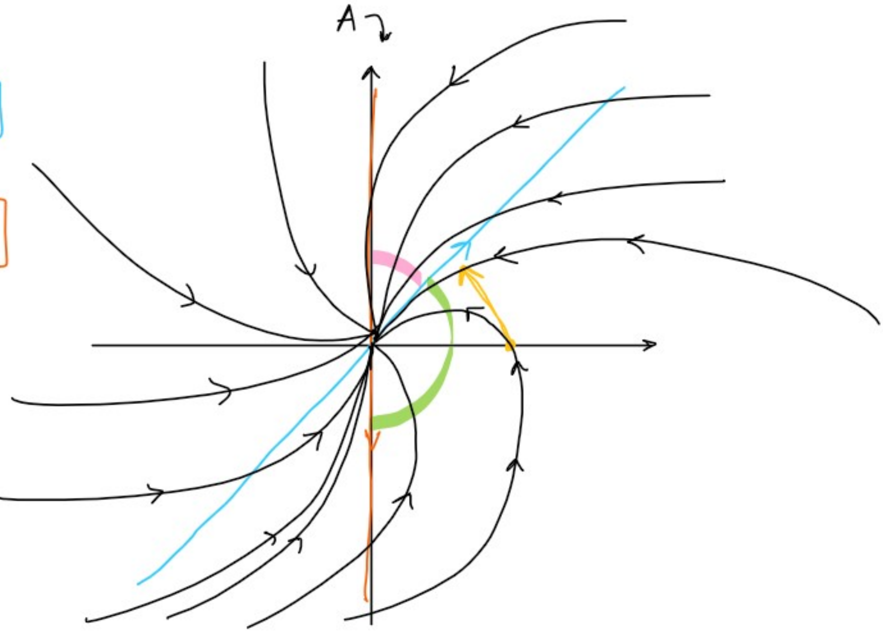


Која је наша? $X=\begin{bmatrix}1\\0\end{bmatrix} \rightsquigarrow X' = \begin{bmatrix}-2 & 1\\-1 & -2\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}-2\\-1\end{bmatrix}$ $\Rightarrow$ лева слика наша

J ↴

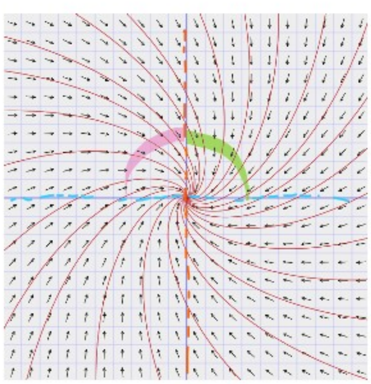


A ↴

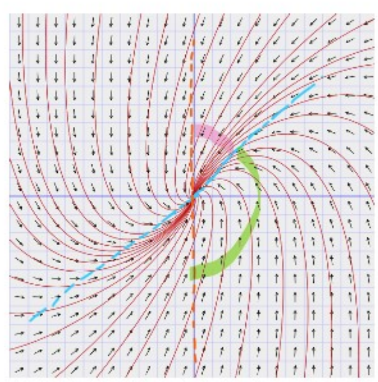


Смер: $X=\begin{bmatrix}1\\0\end{bmatrix} \Rightarrow X' = AX = \begin{bmatrix}-1 & -1\\2 & -3\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}-1\\2\end{bmatrix}$

J ↴



A ↴



3. Скицирати фазни портрет динамичког система $X' = AX$, ако је:

(1) $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$; (2) $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$;

(3) $A = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$; (4) $A = \begin{bmatrix} -3 & 1 \\ -1 & -1 \end{bmatrix}$;

(1) $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ $\lambda_1 = \lambda_2 = 2$

$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \vee \cancel{\begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}}$ $\lambda \neq 0$

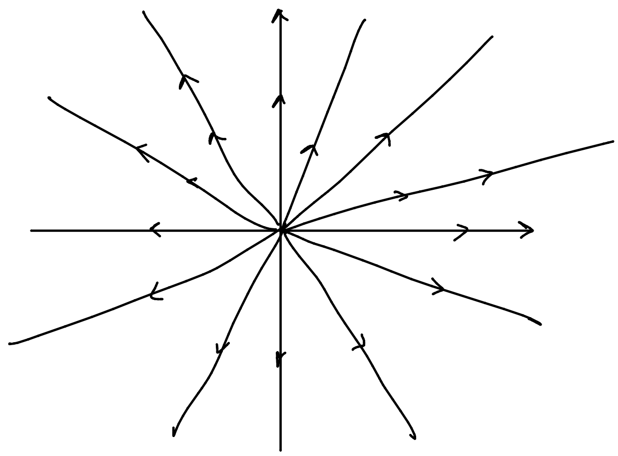
код нас

$\lambda > 0$: нестабилна звезда

$\lambda < 0$: стабилна звезда

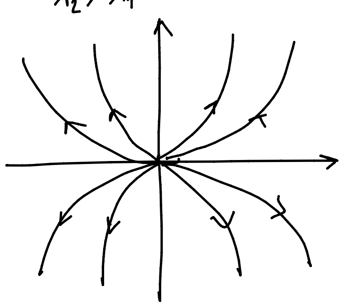
звезда (сингуларни чвор)

$X^* = (0,0)$

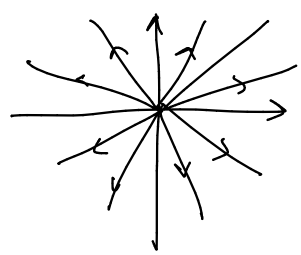


$J = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$:

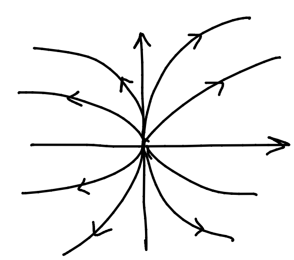
$\lambda_2 > \lambda_1 > 0$



$\lambda_1 = \lambda_2 > 0$



$\lambda_1 > \lambda_2 > 0$



(3) $A = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

$\lambda_1 = \lambda_2 = 1$

$X^* = (0,0)$

Жордан: $\begin{bmatrix} 1 & \\ & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 \\ & 1 \end{bmatrix}$?

$(A - E)\gamma_1 = 0 \dots v = \begin{bmatrix} 1 \end{bmatrix} \to 1$ Жорд. блок $\Rightarrow J = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$(A-E)\gamma_1 = 0 \;\ldots\; \gamma_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \to$ 1 Жорд. блок $\Rightarrow J = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

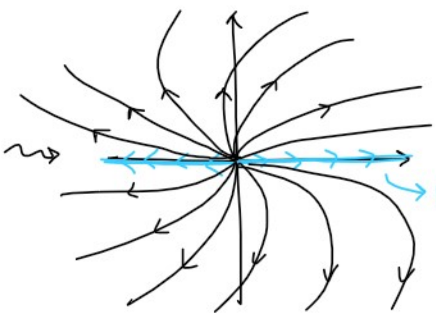
$\begin{bmatrix} \lambda & 1 \\ & \lambda \end{bmatrix} \rightsquigarrow$ дегенерисани чвор

$\lambda \neq 0$

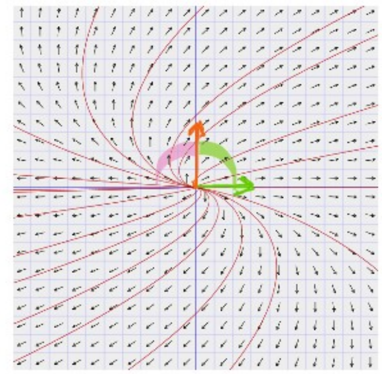
$\lambda > 0$: нестабилан ← код нас

$\lambda < 0$: стабилан

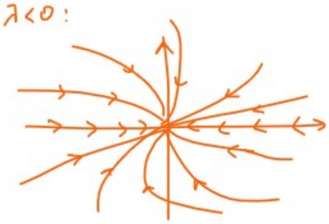
$J = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \rightsquigarrow$



нестабилно јер $\lambda = 1 > 0$



у стабилном случају $J = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$, $\lambda < 0$:



$(A-E)\gamma_2 = \gamma_1$ ← уопштени

$\therefore \gamma_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow T = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

смер: $X = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $X' = AX = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$

чешће

ређе