2. Скицирати фазни портрет динамичког система $X' = AX$, ако је:

(1) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$; (2) $A = \begin{bmatrix} -2 & -5 \\ 2 & 2 \end{bmatrix}$;

(3) $A = \begin{bmatrix} -1 & -1 \\ 2 & -3 \end{bmatrix}$. (4) $A = \begin{bmatrix} 7 & -10 \\ 4 & -5 \end{bmatrix}$.

(2.2) $\lambda_{1/2} = \pm i\sqrt{6} \longrightarrow$ центар $\rightarrow$ елипсе око коорд. почетка (спец. у (2.1) су кругови са центром у (0,0))

$(\alpha \pm i\beta \wedge \alpha = 0 \Rightarrow \pm i\beta)$

$$A = PDP^{-1}$$

$$D = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix} = \begin{bmatrix} 0 & \sqrt{6} \\ -\sqrt{6} & 0 \end{bmatrix}, \quad P = \begin{bmatrix} Re(v)\downarrow & Im(v)\downarrow \end{bmatrix}$$

v – компл. сопс. вектор за $\alpha + i\beta$

$\lambda_1 = \overline{\lambda_2} = i\sqrt{6}$

$(A - \lambda_1 E)v_1 = 0$

$\lambda_1 \rightsquigarrow v_1$

$\lambda_2 \rightsquigarrow v_2$

$$\begin{bmatrix} -2-i\sqrt{6} & -5 \\ 2 & 2-i\sqrt{6} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = 0, \quad a, b \in \mathbb{C}$$

$\lambda_2 = \overline{\lambda_1} \Rightarrow v_2 = \overline{v_1}$

$$(-2-i\sqrt{6})a - 5b = 0$$
$$\cancel{2a + (2-i\sqrt{6})b = 0}$$

$$b = \frac{-2-i\sqrt{6}}{5} a$$

$$a = -5,\ b = 2+i\sqrt{6} \Rightarrow v_1 = \begin{bmatrix} -5 \\ 2+i\sqrt{6} \end{bmatrix}$$

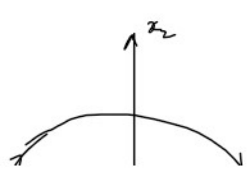
$$Re(v) = \begin{bmatrix} -5 \\ 2 \end{bmatrix}, \quad Im(v) = \begin{bmatrix} 0 \\ \sqrt{6} \end{bmatrix}$$

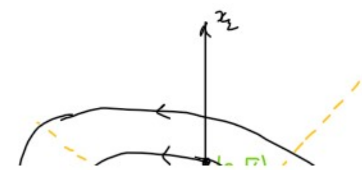
$$P = \begin{bmatrix} -5 & 0 \\ 2 & \sqrt{6} \end{bmatrix}$$

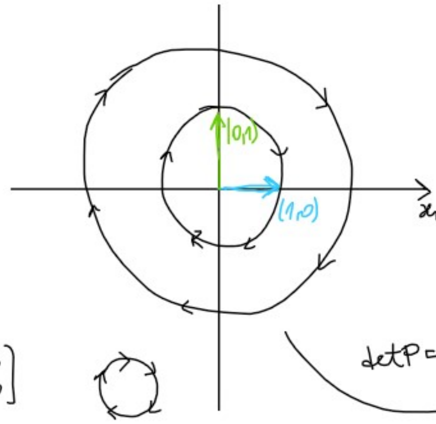
$$X' = DX \Rightarrow \begin{aligned} x_1' &= x_2\sqrt{6} \\ x_2' &= -x_1\sqrt{6} \end{aligned} \rightsquigarrow \begin{aligned} x_1 &= c_1\cos(\sqrt{6}t) + c_2\sin(\sqrt{6}t) \\ x_2 &= -c_1\sin(\sqrt{6}t) + c_2\cos(\sqrt{6}t) \end{aligned}$$

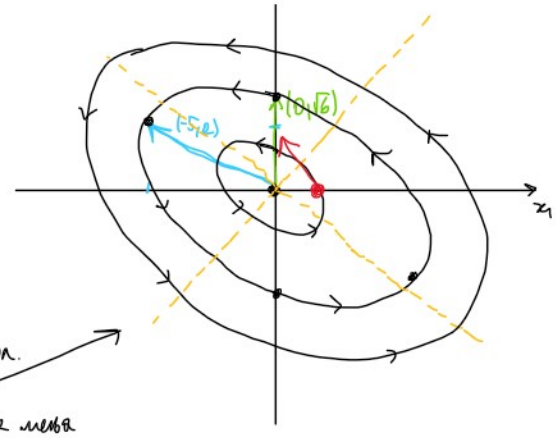
$$\|X\|^2 = x_1^2 + x_2^2 = \underline{c_1^2\cos^2(\sqrt{6}t)} + \underline{c_2^2\sin^2(\sqrt{6}t)} + \underline{c_1^2\sin^2(\sqrt{6}t)} + \underline{c_2^2\cos^2(\sqrt{6}t)} = c_1^2 + c_2^2 \equiv const$$

$\Rightarrow$ решења су на кругу







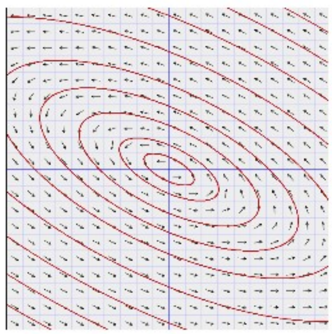


$\det P = -5\sqrt{6} < 0 \to$ рефл.

→ смер се мења

$D = \begin{bmatrix} 0 & \sqrt{6} \\ -\sqrt{6} & 0 \end{bmatrix}$

$D = \begin{bmatrix} 0 & -\sqrt{6} \\ \sqrt{6} & 0 \end{bmatrix}$



$X = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$X' = AX = \begin{bmatrix} -2 & -5 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

(2.3) $A = \begin{bmatrix} -1 & -1 \\ 2 & -3 \end{bmatrix}$

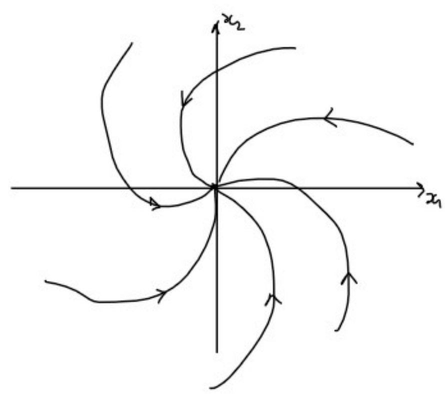
$\lambda_{1/2} = -2 \pm i \quad (\alpha \neq 0)$

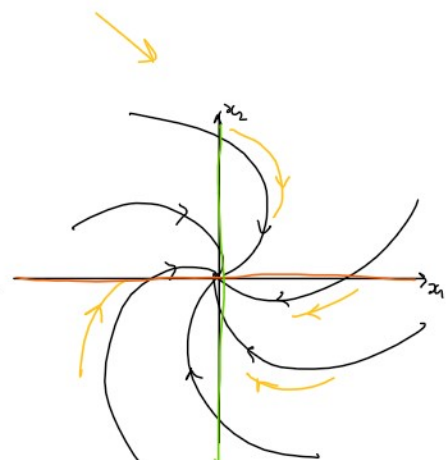
$(-2 < 0)$

стабилна спирала

$D = \begin{bmatrix} -2 & 1 \\ -1 & -2 \end{bmatrix}$ $\quad 1 > 0$

негативан смер





$\lambda = \alpha + i\beta = -2 + i$

$(A - \lambda E)v = 0$

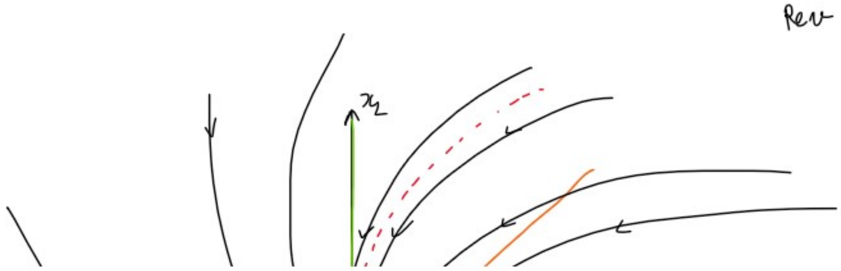
$v = \begin{bmatrix} 1 \\ 1-i \end{bmatrix} \rightsquigarrow P = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$

Re v Im v

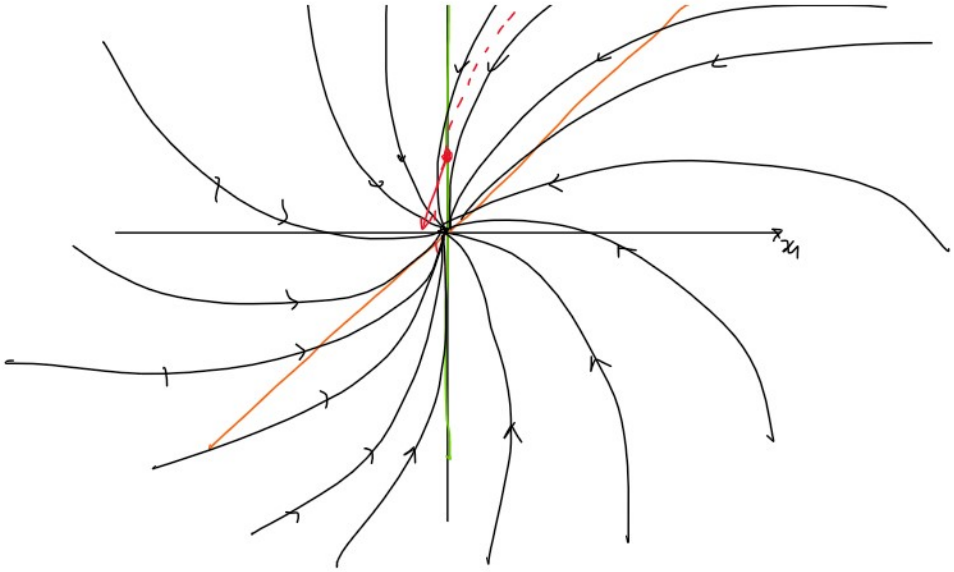
$\det P = -1 < 0$

↓

мења оријентацију

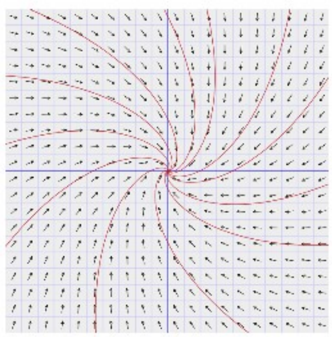


$X = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad X' = AX = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$

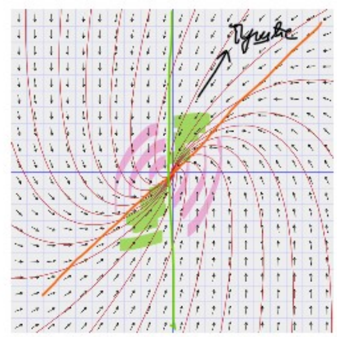


$X = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad X' = AX = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$

D



A



3. Скицирати фазни портрет динамичког система $X' = AX$, ако је:

(1) $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$;

(2) $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$;

(3) $A = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$;

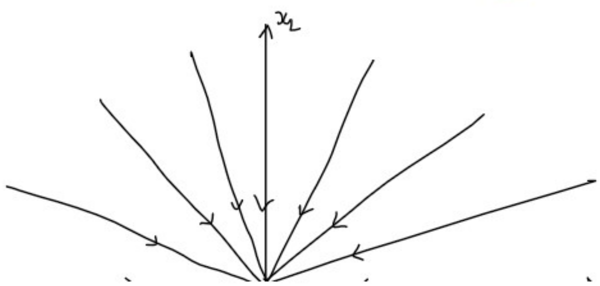
(4) $A = \begin{bmatrix} -3 & 1 \\ -1 & -1 \end{bmatrix}$;

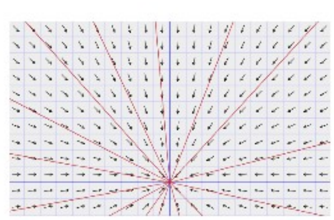
(3.2) $\lambda_1 = \lambda_2 = -1 < 0$

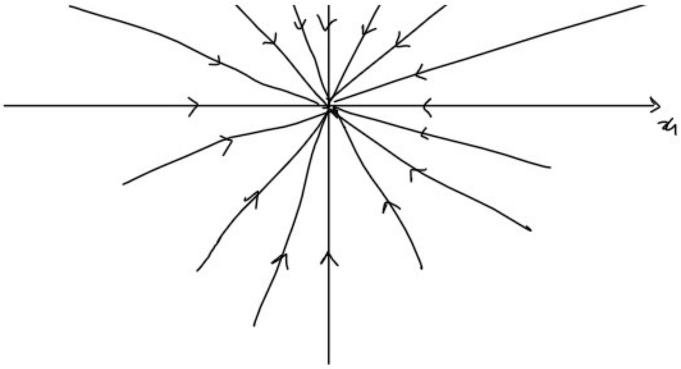
$A \sim \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad \overset{?}{\vee} \quad A \sim \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$

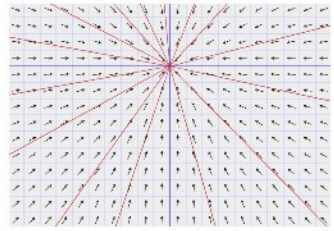
$-1 < 0$

стабилна звезда



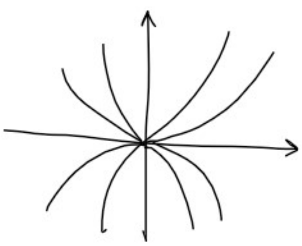




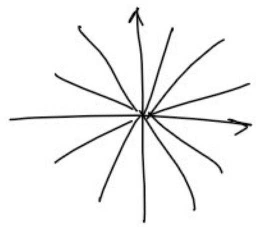


$$\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}$$

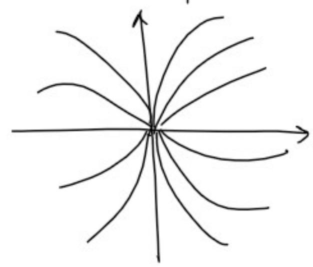
$\lambda_1 < \lambda_2$



$\lambda_1 = \lambda_2$



$\lambda_2 < \lambda_1$



(3.4) $A = \begin{bmatrix}-3 & 1\\ -1 & -1\end{bmatrix}$ $\quad X^* = (0,0)$ $(\det A \neq 0)$

$\lambda = \lambda_1 = \lambda_2 = -2$

$$A \sim \begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix} \quad \lor \quad \underbrace{A \sim \begin{bmatrix}-2 & 1\\ 0 & -2\end{bmatrix}}$$

ако је димензија сопственог потпростора од λ мања од 2 (једнака 1)

$(A - \lambda E)v = 0$

$$\begin{bmatrix}-1 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = 0 \Rightarrow -\alpha + \beta = 0 \rightarrow v = \begin{bmatrix}1\\ 1\end{bmatrix}$$

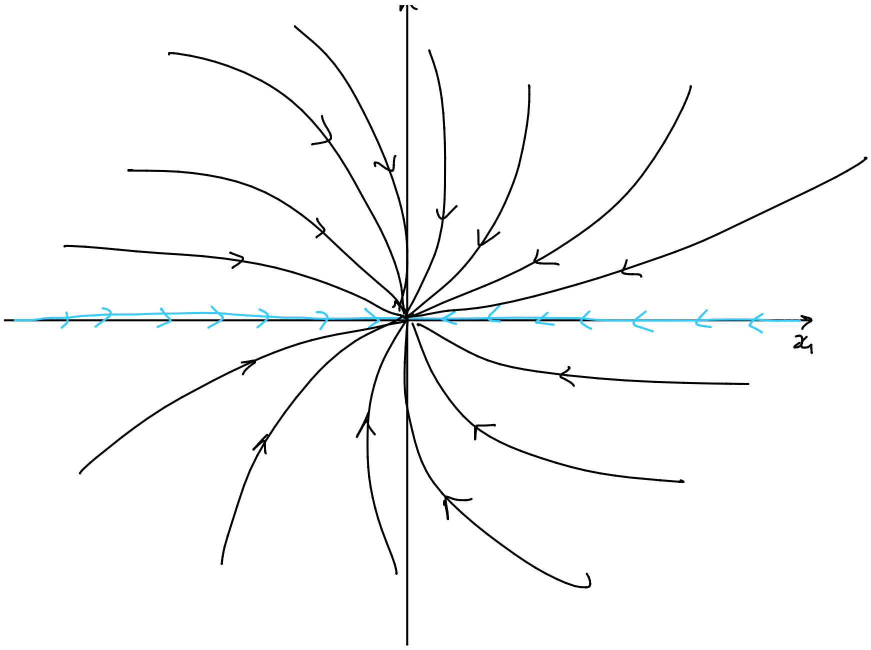
$\alpha = \beta$

$\dim\left(Lin\left(\begin{bmatrix}1\\ 1\end{bmatrix}\right)\right) = 1 \Rightarrow$ један Жорданов блок

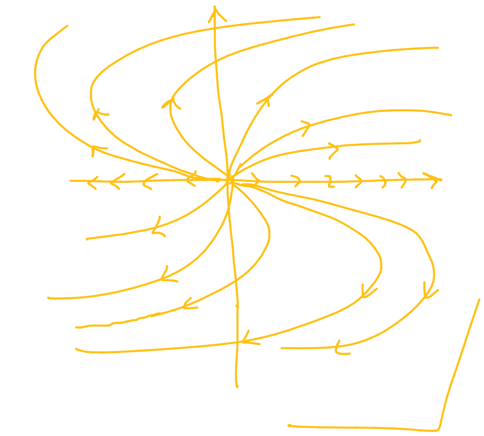
$A \sim \begin{bmatrix}-2 & 1\\ 0 & -2\end{bmatrix} = D \rightarrow$ стабилан дегенерисан чвор

$(-2 < 0)$

У случају: $D = \begin{bmatrix}2 & 1\\ 0 & 2\end{bmatrix}$ нест.



У случају: $D=\begin{bmatrix}2 & 1\\ 0 & 2\end{bmatrix}$ нест.



$P = ?$ → тражимо v_2 генералисани сопствени вектор за $v_1 = v$

$$(A-\lambda E)v_2 = v_1$$

$$\begin{bmatrix}-1 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}\gamma\\ \delta\end{bmatrix} = \begin{bmatrix}1\\ 1\end{bmatrix}$$

$$-\gamma+\delta=1 \Rightarrow \delta = 1+\gamma$$

нпр. $\left.\begin{array}{l}\gamma=0\\ \delta=1\end{array}\right\}\ v_2 = \begin{bmatrix}0\\ 1\end{bmatrix}$

$$P = \begin{bmatrix}v_1\downarrow & v_2\downarrow\end{bmatrix} = \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix},$$ $\det P = 1 > 0$ нема рефл.