(1) Наћи реш. ДЈ $x'\sin t - x\cos t = -\frac{\sin^2 t}{t^2}$ тд. $\lim_{t\to\infty} x(t) = 0$.

← није Кошијев услов

I начин: формула

II начин: $x'\sin t - x\cos t$

$$\left(\frac{x}{\sin t}\right)' = \frac{x'\sin t - x\cos t}{\sin^2 t} \Rightarrow \left(\frac{x}{\sin t}\right)' = -\frac{1}{t^2} \Big/ \int dt$$

$$\frac{x}{\sin t} = \frac{1}{t} + C$$

$$x = \frac{\sin t}{t} + C\sin t, \quad C\in\mathbb{R}$$

$$\lim_{t\to\infty}\left(\boxed{\frac{\sin t}{t}} + C\sin t\right) = 0$$

0 $\quad 0$ → једино: $C = 0$ $\qquad$ ПР: $x = \frac{\sin t}{t}$.

5 Бернулијева ДЈ

$$x' + p(t)x = q(t)x^\alpha, \quad p,q:(a,b)\to\mathbb{R} \text{ непр.} \quad \alpha\in\mathbb{R}$$

$\alpha\in\{0,1\}$ – лин

смена: $y(t) = x(t)^{1-\alpha}$

$y' = (x^{1-\alpha})' = (1-\alpha)x^{-\alpha}\cdot x'$ $\Big\}$ → ЛИН

(2) а) $x' = \frac{x}{t} - x^2$

б) $x' + \frac{x}{t} = x^2\frac{\ln t}{t}$

в) $tx' - 2t\sqrt{x} = 4x$

б) $x^2\frac{\ln t}{t} = q(t)x^\alpha \Rightarrow \alpha = 2 \qquad (t>0)$

$$y = x^{1-\alpha} = x^{1-2} = x^{-1} = \frac{1}{x}$$

$$y' = \underbrace{-\frac{1}{x^2}\cdot x'}$$

$$\Big/ : x^2 \Rightarrow \underbrace{x'\cdot x^{-2}} + \frac{1}{t}\cdot\underbrace{x^{-1}} = \frac{\ln t}{t}$$

$$-y' + \frac{1}{t}y = \frac{\ln t}{t}$$

$$/ : x \quad \Rightarrow \quad \underbrace{x \cdot x} + \frac{1}{t} \cdot \underbrace{x^{-1}} = \frac{\ln t}{t}$$

$$-y' + \frac{1}{t} y = \frac{\ln t}{t}$$

$$y' - \frac{1}{t} y = -\frac{\ln t}{t}$$

$$\vdots$$

$$y(t) = ct + \ln t + 1, \quad c \in \mathbb{R}$$

$$x(t) = \frac{1}{y(t)} = \frac{1}{ct + \ln t + 1}, \quad c \in \mathbb{R}$$

$\boxed{6}$ Рикатијева ДЈ

$$x' = p(t)x^2 + q(t)x + r(t), \quad p, q, r : (a, b) \to \mathbb{R} \text{ непр}$$

$p \equiv 0 \to$ ЛИН

$r \equiv 0 \to$ БЕР $(\alpha = 2)$

смена: x_p – партикуларно реш.

$x(t) \rightsquigarrow y(t)$: $\left.\begin{aligned} x(t) &= x_p(t) + \frac{1}{y(t)} \\ x' &= x_p' - \frac{1}{y^2} \cdot y' \end{aligned}\right\} \rightsquigarrow$ ЛИН

3) а) $t(2t-1)x' + x^2 - (4t+1)x + 4t = 0$

б) $x' = \dfrac{2\sin t - x^2 \sin t \cos^2 t}{\cos^2 t}$

в) $x' + x^2 + \dfrac{4x}{t} + \dfrac{2}{t^2} = 0$

б) $x' = -x^2 \cdot \sin t + 2\dfrac{\sin t}{\cos^2 t}$

$x_p = ?$ $\quad x = \dfrac{1}{\cos t} \longrightarrow x' = -\dfrac{1}{\cos^2 t} \cdot (-\sin t) \to$ добар смер

$x = \dfrac{a}{\cos t}, \quad a = ?$

$$x' = \frac{\sin t}{\cos^2 t} \cdot a = -\frac{a^2}{\cos^2 t} \sin t + 2\frac{\sin t}{\cos^2 t} \quad / : \frac{\sin t}{\cos^2 t}$$

$$a = -a^2 + 2 \Rightarrow a^2 + a - 2 = 0$$

$$a = -a^2 + 2 \Rightarrow a^2 + a - 2 = 0$$

$$(a+2)(a-1) = 0$$

$$x_p = \frac{1}{\cos t} \qquad \left(x_p = \frac{-2}{\cos t} \right)$$

$$x = x_p + \frac{1}{y}$$

$$x' = x_p' - \frac{y'}{y^2} = \frac{\sin t}{\cos^2 t} - \frac{y'}{y^2}$$

$$\frac{\sin t}{\cos^2 t} - \frac{y'}{y^2} = -\left(\frac{1}{\cos t} + \frac{1}{y}\right)^2 \sin t + 2\frac{\sin t}{\cos t}$$

$$\vdots$$

$y' - 2\operatorname{tg} t \cdot y = \sin t \rightsquigarrow$ лин.

$y(t) \rightsquigarrow x(t)$

ОР: $\begin{cases} x_p \\ x_p + \frac{1}{y(t)} \end{cases}$

2) x_p – полином

$x_p = a$

$x_p = at + b$

$x_p = at^2 + bt + c$

$\vdots$

В) $x_p = \frac{a}{t}$ ⋱

Смене: $\boxed{1}$ $x(t) \rightsquigarrow x(u)$

(4) $2t x''(t) + (1+\sqrt{t})\, x'(t) - x(t) = t + \sin(\sqrt{t})$, $t > 0$

увести смену $u = \sqrt{t} \rightsquigarrow \frac{du}{dt} = \frac{1}{2\sqrt{t}} = \frac{1}{2u}$

$t = u^2$

$\frac{dx}{dt} \rightsquigarrow \frac{dx}{du}$

$\frac{d^2x}{dt^2} \rightsquigarrow \frac{d^2x}{du^2}$

$\to x'(t) = \frac{dx}{dt} = \frac{dx}{du} \cdot \frac{du}{dt} = x'_u \cdot \frac{1}{2u}$

$\frac{df}{dt} = \frac{du}{dt} \cdot \frac{df}{du}$

$$x''(t) = \frac{d}{dt}\left(\frac{d}{dt} x(t)\right) = \frac{d}{dt}\left(x'_u \cdot \frac{1}{2u}\right) =$$

$$= \frac{du}{dt} \cdot \frac{d}{du}\left(x'_u \cdot \frac{1}{2u}\right) = \frac{1}{2u} \cdot \left(\frac{d}{du}(x'_u) \cdot \frac{1}{2u} + x'_u \cdot \frac{d}{du}\left(\frac{1}{2u}\right)\right) =$$

$$= \frac{1}{2u} \cdot \left(x''_{uu} \cdot \frac{1}{2u} + x'_u \cdot \frac{1}{2}\left(-\frac{1}{u^2}\right)\right) =$$

$$= \frac{1}{4u^2} x''_{uu} - \frac{1}{4u^3} x'_u$$

$$2u^2\left(\frac{1}{4u^2}x''_{uu} - \frac{1}{4u^3}x'_u\right) + (1+u)\,x'_u\cdot\frac{1}{2u} - x = u^2+\sin u \quad /\cdot 2$$

$$x''_{uu} - \cancel{\frac{x'_u}{u}} + \cancel{\frac{x'_u}{u}} + x'_u - 2x = 2u^2 + 2\sin u$$

$$x''_{uu} + x'_u - 2x = 2u^2 + 2\sin u$$

[2] $x(t) \rightsquigarrow y(t)$

$$y=f(x),\quad y'=f'(x)\cdot x'$$

(5) а) $t\,\boxed{x^2x'} + \boxed{x^3} = t\cos t$

$y=x^3 \to y'=3x^2x' \Rightarrow t\,\frac{y'}{3} + y = t\cos t \Rightarrow y'+\frac{3}{t}\,y = 3\cos t$ (ЛНУ)

б) $\boxed{x'\cos x} = \frac{\boxed{\sin x}}{t} - \boxed{\sin^2 x}$

$y=\sin x \rightsquigarrow y'=\cos x\cdot x' \Rightarrow y' = \frac{y}{t} - y^{2}$ $\quad \alpha=2$ (БЕР, РИК)

в) $\boxed{x'\,\mathrm{tg}\,x} + 4t^3\boxed{\cos^3 x} = 2t$

$\mathrm{tg}\,x = \frac{\sin x}{\cos x}$ $\qquad y=\frac{1}{\cos x} \rightsquigarrow y'=\frac{\sin x}{\cos^2 x}\cdot x' = y\cdot \mathrm{tg}\,x\,x' \Rightarrow \frac{y'}{y} + 4t^3\,\frac{1}{y^3} = 2t \Rightarrow y' + 4t^3\cdot y^{-2} = 2ty$ $\quad \alpha=-2$ (БЕР)

г) $t\boxed{e^x x'} - 2t\boxed{e^{x/2}} = 4\boxed{e^x}$

$y=e^{x/2} \rightsquigarrow t\,y\cdot 2y' - 2t\,y = 4y^2 \Rightarrow y'-1 = 2\,\frac{y}{t}$ (ЛНУ)

$y' = e^{x/2}\cdot\frac{1}{2}\,x'$

[3] $x(t) \rightsquigarrow y(u)$

(6) Решити ДЈ: $\frac{dx}{dt} = \frac{x\left((\ln x)^2 + t\right)}{2t^{3/2}}$, $x>0$, $t>0$

Увести смене: $u=\sqrt{t}$
$y=\ln x$

$$\frac{dx}{dt} \rightsquigarrow \frac{dy}{du}$$

$$\frac{dy}{du} = \boxed{\frac{dy}{dx}}\cdot\frac{dx}{dt}\cdot\boxed{\frac{dt}{du}} \Rightarrow \frac{dy}{du} = e^{-y}\cdot 2u\cdot\frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = \frac{dy}{du}\cdot\frac{e^{y}}{2u}$$

$t=u^2$

$\frac{dt}{du} = \frac{d(u^2)}{du} = 2u$

$y=\ln x$

$\frac{dy}{dx} = \frac{d(\ln x)}{dx} = \frac{1}{x} = e^{-y}$

$\Rightarrow \frac{\quad}{dt} = \frac{\quad}{du} \cdot \frac{\quad}{2u}$

$y = \ln x$

$\frac{dy}{dx} = \frac{d(\ln x)}{dx} = \frac{1}{x} = e^{-y}$

$x = e^{y} \Rightarrow \frac{1}{x} = e^{-y}$

$\frac{dt}{du} = \frac{d(u^2)}{du} = 2u$

$$y' \cdot \frac{e^{y}}{2u} = \frac{e^{y}(y^2+u^2)}{2u^3} \Rightarrow y' = \frac{y^2+u^2}{u^2} = \left(\frac{y}{u}\right)^2 + 1 \; \ldots \; (\text{xom})$$

$y' \swarrow \frac{dy}{du}$

$$y(u) \rightsquigarrow x(t) = e^{y(t)} = e^{y(u^2)}$$