$$x_1' = 6x_1 - x_2$$
$$x_2' = 4x_1 + 2x_2 + e^{4t}\sqrt{t}$$

$$x'' - 8x' + 16x + e^{4t}\sqrt{t} = 0$$

$$x_2 = 6x_1 - x_1'$$

$$(6x_1 - x_1')' = 4x_1 + 2(6x_1 - x_1') + e^{4t}\sqrt{t}$$

$$6x_1' - x_1'' = 4x_1 + 12x_1 - 2x_1' + e^{4t}\sqrt{t}$$

$$x_1'' - 8x_1' + 16x_1 + e^{4t}\sqrt{t} = 0$$

$$B(t) = \begin{bmatrix} 0 \\ e^{4t}\sqrt{t} \end{bmatrix}$$

$$y_p(t) = e^{tA} \cdot \int e^{-tA} \cdot B(t)\,dt$$

ОР: $\boxed{x_1(t) = \ldots}$

$x_2(t) = \ldots$

$$\left.\begin{aligned} x' &= 6x - 2y \\ y' &= 5x \\ z' &= z^4 + z \end{aligned}\right\} \quad A = \begin{bmatrix} 6 & -2 \\ 5 & 0 \end{bmatrix}$$

нелинеарно

Бернулијева: $z' + p(t)z = g(t)z^{\alpha}$ $\quad \alpha = 4$

нпр. $x(t) = c_1\cos t + c_2 \sin t$

$y(t) = -c_2 \cos t$

$z(t) = (1+t^3)c_3 + t$

$c_1(s),\ c_2(s),\ c_3(s)$

$$s = x(0) = c_1(s)\cdot\cos 0 + c_2(s)\cdot \sin 0$$

$$\frac{3}{2}s = y(0) = -c_2 \cos 0$$

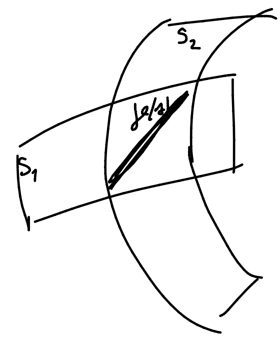
$$\sqrt[3]{\frac{2}{s-2}} = z(0) = 1\cdot c_3(s) + 0$$

$\vdots$

$c_1(s), c_2(s), c_3(s)$

$S_1: 3x = 2y$

$S_2: z^3(x-2) = 2$



$y = \frac{3}{2}x\ ,\ z = \sqrt[3]{\frac{2}{x-2}}$

$$\gamma(s) = \left(s, \frac{3}{2}s, \sqrt[3]{\frac{2}{s-2}}\right)$$

2. Дат је нелинеаран систем диференцијалних једначина

$$x_1' = 4x_1^2x_2 - x_1E(x_1,x_2)$$
$$x_2' = -2x_1^3 - x_2E(x_1,x_2),$$

где је $E(x_1,x_2) = x_1^2 + 2x_2^2 - 4$.

(а) Испитати стабилност еквилибријума $(0,0)$.

(б) Одредити све еквилибријуме датог система. За које од тих еквилибријума се може користити функција $E(x_1,x_2)$ као функција Љапунова за одређивање стабилности? Образложити одговор.

а) • дефиниција X

• сопс. вр.

• ф-ја Љапунова

$$F(x_1,x_2) = \begin{bmatrix} \overbrace{4x_1^2x_2 - x_1^3 - 2x_1x_2^2 + 4x_1}^{F_1} \\ -2x_1^3 - x_1^2x_2 - 2x_2^3 + 4x_2 \end{bmatrix}$$

$$dF(x_1,x_2) = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} \end{bmatrix}$$

$$dF(0,0) = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \quad \lambda_1 = \lambda_2 = 4 > 0 \quad \text{нестабилан!}$$

б)
$$\left.\begin{aligned} 4x_1^2x_2 - x_1E &= 0 \\ -2x_1^3 - x_2E &= 0 \end{aligned}\right\}$$

$x_1(\ldots)$

1° $x_1 = 0 \Rightarrow x_2E = 0 \Rightarrow \boxed{(0,0),\ (0,\sqrt{2}),\ (0,-\sqrt{2})}$

2° $4x_1x_2 - E = 0$

$E = 4x_1x_2$

$-2x_1^3 - x_2 \cdot 4x_1x_2 = 0$

$x_1(-2x_1^2 - 4x_2^2) = 0 \to x_1 = 0$ ✓

$2x_1^2 + 4x_2^2 = 0 \to x_1 = x_2 = 0$ ✓

$E(x_1,x_2) = x_1^2 + 2x_2^2 - 4$

1° $E \in C^1$ ✓

$\boxed{2^\circ\ E(X^*) = 0,\quad E(X) > 0,\quad X \in \mathcal{U} \setminus \{X^*\}}$

3° ... ← не треба

$E(0,0) = 0 + 0 - 4 = -4 \neq 0$ ↯

$E(0,\sqrt{2}) = E(0,-\sqrt{2}) = 0$

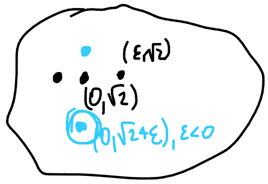
$E(0,\sqrt{2}+\varepsilon) = 0^2 + 2\cdot(\sqrt{2}+\varepsilon)^2 - 4 = 2\cdot(2+\varepsilon^2+2\sqrt{2}\varepsilon) - 4 = 2\varepsilon^2 + 4\sqrt{2}\varepsilon = 2\varepsilon(\varepsilon + 2\sqrt{2}) < 0$

$\varepsilon \to 0$

($\varepsilon < 0$ и $\varepsilon \to 0$)

$\varepsilon \to 0$

$\boxed{\varepsilon < 0 \text{ и } \varepsilon \to 0}$



$E(0,-\sqrt{2}+\varepsilon) = \dots < 0$, $\varepsilon \to 0$, $\varepsilon > 0$

E није фја Љап. ни за један еквил.

$y' = e^{x} \cdot Ay \quad / \cdot e^{-x}$

$e^{-x} \cdot y' = Ay$

$f(x) = e^{-x}$

$\frac{dx}{e^{-x}} = dt \quad / \int$

$t = \int e^{x} dx = e^{x}$

$\boxed{t = e^{x}}$

$f(x) \cdot y' = Ay$

смена: $x \rightsquigarrow t$, $\underline{x(t)}$

$\boxed{f(x)} \cdot \frac{dy}{dx} = \frac{dy}{dt} = \frac{dy}{dx} \cdot \boxed{\frac{dx}{dt}}$

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$\frac{dx}{dt} = f(x) \Rightarrow \frac{dx}{f(x)} = dt \quad / \int$

$e^{-x} y' = Ay \rightsquigarrow y' = Ay$ ✓

$\searrow$ по t

---

3. Решити систем диференцијалних једначина

$$y_1' = \frac{x(y_1+y_2)}{y_1^2+y_2^2}, \quad y_2' = \frac{x(y_1-y_2)}{y_1^2+y_2^2}.$$

$\rightarrow \frac{dy_1}{x(y_1+y_2)} = \frac{dy_2}{x(y_1-y_2)} = \frac{dx}{y_1^2+y_2^2} \dots$

$\rightarrow \frac{y_1'}{y_2'} = \frac{dy_1}{dy_2} = \frac{y_1+y_2}{y_1-y_2} = \frac{\frac{y_1}{y_2}+1}{\frac{y_1}{y_2}-1}$

$y' = \frac{a_1 y + b_1 x + \cancel{c_1}^{\,0}}{a_2 y + b_2 x + \cancel{c_2}^{\,0}}$

$y_1 = y_1(y_2)$

$$A^2 \cdot e^A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad /\det \qquad A = ?$$

$$\det(A^2) \cdot \det(e^A) = \det\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right)$$

$$\underbrace{\left(\det(A)\right)^2}_{\geq 0} \cdot \underbrace{e^{trA}}_{>0} = \underbrace{-2}_{<0} \quad ↯$$

0, 0, $\underline{2}$ ✓