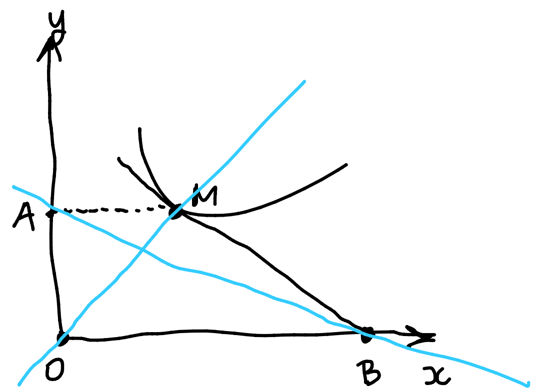
1



$x(t), y(t)$

$y(x)$

$M(x_0, y(x_0))$

$O(0,0)$

$A(0, y(x_0))$

$MB: k = y'(x_0)$ $B(x_B, 0)$

$y = y'(x_0)\cdot x + n \leftarrow (x_0, y(x_0))$

$y(x_0) = y'(x_0)\cdot x_0 + n \Rightarrow n = \ldots$

$0 = y'(x_0)\cdot x_B + y(x_0) - y'(x_0)\cdot x_0$

$x_B = \ldots$

$OM: k_1$
$AB: k_2$ $\Big\}$ $k_1 \cdot k_2 = -1$

$k_1 = \dfrac{y_M - y_0}{x_M - x_0} = \ldots$

$k_2 = \dfrac{y_A - y_B}{x_A - x_B} = \ldots$

$y(2) = 2$

$\ldots\ y' = \dfrac{xy}{y^2 - x^2}$ $\quad :y^2$ (хомогена)

$y' = \dfrac{\frac{x}{y}}{1 - \left(\frac{x}{y}\right)^2} \ldots$

2 $\quad X' = AX \quad , \quad X(t) = e^{2t}\cdot v, \quad v \in \mathbb{R}^3 \setminus \{\vec{0}\} \ \ldots \ \boxed{\alpha = 1}$

$X'(t) = A X(t)$

$\rightarrow \boxed{2v = Av}$

$$\begin{bmatrix} \alpha & 3 & \alpha \\ -1 & 2\alpha+1 & 1 \\ 0 & -2 & 2 \end{bmatrix}$$

$X' = AX$

$2, \ 2 \pm 2i$

$$J = \begin{bmatrix} 2 & & \\ & 2 & 2 \\ & -2 & 2 \end{bmatrix}$$

$(A - 2E)v = 0 \ \ldots \ v = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$$(A-2E)v=0 \;\dots\; v=\begin{bmatrix}1\\0\\1\end{bmatrix}$$

$$2+2i: \quad (A-(2+2i)E)v_1=0 \;\dots \qquad T=\begin{bmatrix} v & \operatorname{Re} v_1 & \operatorname{Im} v_1\end{bmatrix} \;\dots$$

③ $x'(t)=x(t)-t+1\,,\ x(0)=1$ $\qquad x_0=1,\ t_0=0$

$x_0(t)\equiv 1$

$$x_1(t)=x_0+\int_0^t (x_0(s)-s+1)\,ds=\underset{1}{\underset{\parallel}{x_0}}+\left(2s-\frac{s^2}{2}\right)\Big|_0^t=1+2t-\frac{t^2}{2}$$

$$x_2(t)=1+\int_0^t (x_1(s)-s+1)\,ds=1+\int_0^t\left(2+s-\frac{s^2}{2}\right)ds=1+2t+\frac{t^2}{2}-\frac{t^3}{6}$$

$\vdots$

$$x_n(t)=1+2t+\frac{t^2}{2}+\frac{t^3}{6}+\dots+\frac{t^n}{n!}-\underbrace{\frac{t^{n+1}}{(n+1)!}}_{\to 0}\longrightarrow t+e^t$$

(индукцией !)

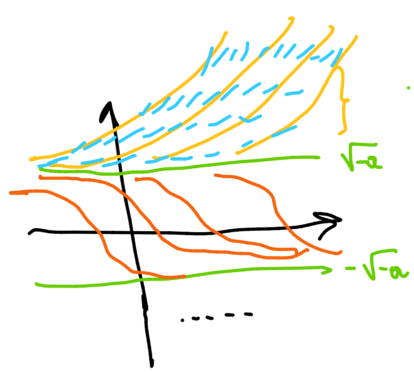
$x'=a+x^2$

1) $a<0 \rightsquigarrow x^2+a=0 \Rightarrow x=\pm\sqrt{-a}$

2) $a=0 \rightsquigarrow x\equiv 0$

3) $a>0 \rightsquigarrow$ нема

1)

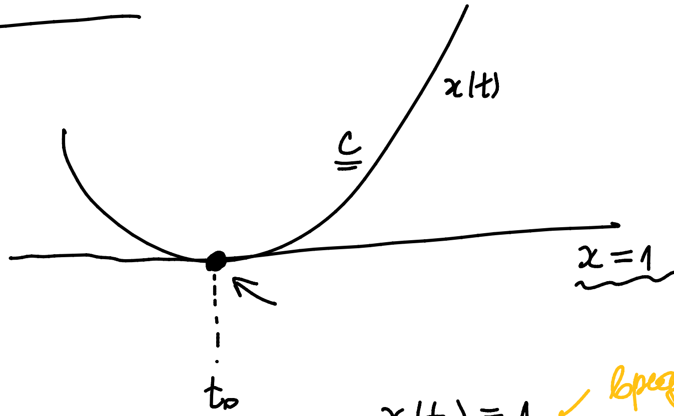


$x'=x^2+a=(x-\sqrt{-a})(x+\sqrt{-a})$

$x>\sqrt{-a}: \ x'>0 \Rightarrow x\nearrow$

$\sqrt{-a}>x>-\sqrt{-a}: \ x'<0\,,\ x\searrow$

⊛ $\underset{\sim}{x=1}$



$x(t_0)=1$ ↙ вредности

$x'(t_0)=0$ ↙ нагиб праве

⊛ $X' = e^t A X$ $\qquad X = \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix}$

$\tau = e^t$

$x_k(t) \rightsquigarrow x_k(\tau)$

$$\frac{dx_k}{dt} = \frac{dx_k}{d\tau} \cdot \underbrace{\frac{d\tau}{dt}}_{= e^t} \Rightarrow \frac{dx_k}{d\tau} = \frac{dx_k}{dt} \cdot e^{-t}$$

$$\Rightarrow \frac{dX}{dt} = \frac{dX}{d\tau} \cdot e^t$$

$$\frac{dX}{dt} = e^t \cdot AX$$

$$\frac{dX}{d\tau} \cdot \cancel{e^t} = \cancel{e^t} AX$$

$$\frac{dX}{d\tau} = AX \rightsquigarrow \text{ОР из (2)}$$

⊛

$$(x_3x_4 - x_2)\frac{\partial u}{\partial x_1} - (2x_1x_2x_4 + x_3)\frac{\partial u}{\partial x_2} + 2x_1(x_2 - x_3x_4)\frac{\partial u}{\partial x_3} + (2x_1x_4^2 + 1)\frac{\partial u}{\partial x_4} = 0$$

$x_1' = x_3x_4 - x_2$

$x_2' = -2x_1x_2x_4 - x_3$

$x_3' = 2x_1(x_2 - x_3x_4)$

$x_4' = 2x_1x_4^2 + 1$

$$\frac{x_3'}{x_1'} = \frac{2x_1(x_2 - x_3x_4)}{x_3x_4 - x_2} = -2x_1$$

$$\hookrightarrow \frac{dx_3}{dx_1} = -2x_1 \Rightarrow$$

$$\boxed{x_3 = -x_1^2 + C_1}$$

$$\boxed{x_3 = -x_1^2 + C_1}$$

$$x_1 x_2' + x_2 x_1' = -x_3 x_1 + x_2 = -x_1'$$

$$(x_2 x_1)' + x_1' = 0 \Rightarrow \boxed{x_2 x_1 + x_1 = C_2}$$

$$2x_1 x_1' + x_3' = -2x_1x_2 + 2x_1x_2 = 0$$

$$\boxed{x_1^2 + x_3 = C_3}$$

$$x_2' + x_1 x_3' = -x_3 - 2x_1x_1x_1^2$$
$$= x_3(-1 - 2x_1x_1^2)$$
$$= -x_3 x_1'$$

$$x_2' + (x_3 x_1)' = 0$$

$$\boxed{x_2 + x_3 x_1 = 0}$$

$$u = \varphi(\psi_1, \psi_2, \psi_3)$$

$$u\big|_{x_1=0} = \frac{x_2 + x_3 x_1}{x_3^2}$$

$$\frac{x_2 + x_3 x_1}{x_3^2} = \varphi(x_3,\ x_2 x_1,\ x_2 + x_3 x_1)$$

$$\varphi(x, y, z) = \frac{z}{x^2}$$

$$u = \frac{\psi_3}{\psi_1^2} = \frac{x_2 + x_3 x_1}{(x_1^2 + x_3)^2}.$$