# $\boxed{6}$ Рикатијева дј.

$$x' = p(t)x^2 + q(t)x + r(t) \qquad p, q, r : (a, b) \to \mathbb{R} \text{ непр.}$$

$p \equiv 0$ : лин

$r \equiv 0$ : БЕР ($\alpha = 2$)

смена: $x_p(t)$ – партикуларно решење

$$x(t) = \underline{\underline{x_p}}(t) + \frac{1}{y(t)} \qquad x(t) \rightsquigarrow y(t)$$

$\hookrightarrow$ лин.

$$x' = x_p' + \left(\frac{1}{y}\right)' = x_p' - \frac{1}{y^2} \cdot y'$$

$x_p$: $x_p' = p x_p^2 + q x_p + r$

$$x_p' - \frac{y'}{y^2} = p \cdot \left(x_p + \frac{1}{y}\right)^2 + q \cdot \left(x_p + \frac{1}{y}\right) + r$$

$$\underbrace{x_p'}_{=} - \frac{y'}{y^2} = \underbrace{\left(p x_p^2 + q x_p + r\right)}_{=} + \frac{2p x_p}{y} + \frac{p}{y^2} + \frac{q}{y} \quad / \cdot y^2$$

$$-y' = 2p x_p y + p + q y$$

$$y' + y(2p x_p + q) = -p \to \text{лин}$$

$$\vdots$$

$$\text{ор: } \begin{cases} x(t) \\ x_p(t) \end{cases}$$

(1) а) $t(2t-1)x' + x^2 - (4t+1)x + 4t = 0$

б) $x' = \dfrac{2\sin t - x^2 \sin t \cos^2 t}{\cos^2 t}$

в) $x' + x^2 + \dfrac{4x}{t} + \dfrac{2}{t^2} = 0$

в) $x_p = ?$

$$\left.\begin{aligned} & x' \sim x^2 \sim \frac{4x}{t} \sim \frac{2}{t^2} \\ & \left(\frac{1}{t}\right)' = -\frac{1}{t^2} \end{aligned}\right\} \frac{1}{t} \rightsquigarrow x_p = \frac{a}{t}, \quad a \in \mathbb{R}$$

$$x_p' + x_p^2 + \frac{4x_p}{t} + \frac{2}{t^2} = 0$$

$$-\frac{a}{t^2} + \frac{a^2}{t^2} + \frac{4a}{t^2} + \frac{2}{t^2} = 0 \quad / \cdot t^2$$

$$-a + a^2 + 4a + 2 = 0$$

$$a^2+3a+2=0$$
$$(a+1)(a+2)=0 \longrightarrow a=-1 \ \vee \ \underline{\underline{a=-2}}$$

$$x_p = -\frac{2}{t}$$

$$x = -\frac{2}{t} + \frac{1}{y}$$

$$x' = \frac{2}{t^2} - \frac{y'}{y^2}$$

$$\frac{2}{t^2} - \frac{y'}{y^2} + \frac{4}{t^2} - \frac{4}{ty} + \frac{1}{y^2} - \frac{8}{t^2} + \frac{4}{yt} + \frac{2}{t^2} = 0 \quad / \cdot y^2$$

$$-y'+1=0$$

$$y'=1 \Rightarrow y = t+c, \ c\in\mathbb{R}$$

$$x = -\frac{2}{t} + \frac{1}{t+c}$$

$$\text{ОР}: \begin{cases} -\frac{2}{t} + \frac{1}{t+c}, \ c\in\mathbb{R} \\ -\frac{2}{t} \end{cases}$$

а) $t(2t-1)\,x' + x^2 - (4t+1)x + 4t = 0$

полином по $t$

$\Rightarrow$ $x_p$ - полином

$x_p = a$

$\to x_p = at+b$

$\hookrightarrow x_p = at^2+bt+c$

$\vdots$

б) $$x' = \frac{2\sin t - x^2 \sin t \cos^2 t}{\cos^2 t} = \frac{2\sin t}{\cos^2 t} - x^2 \sin t$$

$$\left(\frac{1}{\cos t}\right)' = -\frac{1}{\cos^2 t}\cdot(-\sin t) = \frac{\sin t}{\cos^2 t}$$

$$x_p = \frac{a}{\cos t}$$

## Смене

$\boxed{1}$ $x(t) \rightsquigarrow x(u)$

(2) $2t x''(t) + (1+\sqrt{t})\,x'(t) - x(t) = t + \sin(\sqrt{t}), \ t>0$

Доказати да се сменом $u=\sqrt{t}$ своди на $x''+x'-2x = 2u^2 + 2\sin u$.

$$x(t) \rightsquigarrow x(u)$$

$$\frac{dx}{dt} \rightsquigarrow \frac{dx}{du}$$

$$\frac{d^2x}{dt^2} \rightsquigarrow \frac{d^2x}{du^2}$$

$$u=\sqrt{t}$$

$$t=u^2$$

$$\frac{du}{dt} = \frac{d(\sqrt{t})}{dt} = \frac{1}{\dots} = \frac{1}{\dots}$$

$$\frac{df}{dt} = \frac{du}{dt}\cdot\frac{df}{du}$$

$$\frac{dx}{dt} = \frac{dx}{du}\cdot\frac{du}{dt} = x'_u \cdot \frac{1}{2u}$$

$$\frac{d}{dt}(x'_u)\cdot\frac{1}{2u} + x'_u \cdot \frac{d}{dt}\left(\frac{1}{2u}\right)$$

$$\frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d}{dt}\left(x'_u\cdot\frac{1}{2u}\right) =$$

$$= \frac{du}{dt}\cdot\frac{d}{du}\left(x'_u\cdot\frac{1}{2u}\right) = \frac{1}{2u}\cdot\left(\frac{d}{du}(x'_u)\cdot\frac{1}{2u} + x'_u\cdot\frac{d}{du}\left(\frac{1}{2u}\right)\right) =$$

$t = u^2$

$$\frac{du}{dt} = \frac{d(\sqrt{t})}{dt} = \frac{1}{2\sqrt{t}} = \frac{1}{2u}$$

$$= \frac{du}{dt}\cdot\frac{d}{du}\left(x'_u\cdot\frac{1}{2u}\right) = \frac{1}{2u}\cdot\left(\frac{d}{du}(x'_u)\cdot\frac{1}{2u} + x'_u\cdot\frac{d}{du}\left(\frac{1}{2u}\right)\right) =$$

$$= \frac{1}{2u}\left(x''_{uu}\cdot\frac{1}{2u} + x'_u\cdot\left(-\frac{1}{2u^2}\right)\right) = \frac{x''_{uu}}{4u^2} - \frac{x'_u}{4u^3}$$

$$2u^2\cdot\left(\frac{x''_{uu}}{4u^2} - \frac{x'_u}{4u^3}\right) + (1+2u)\cdot x'_u\cdot\frac{1}{2u} - x = u^2 + \sin u$$

$$\frac{x''_{uu}}{2} \underbrace{- \frac{x'_u}{2u}}_{\times} + \underbrace{\frac{x'_u}{2u}}_{\times} + \frac{x'_u}{2} - x = u^2 + \sin u \,/2$$

$$x''_{uu} + x'_u - 2x = 2u^2 + 2\sin u$$

$y = f(x) \rightsquigarrow y' = f'(x)\cdot x'$

$\boxed{2}$ $x(t) \rightsquigarrow y(t)$

(3) а) $t\boxed{x^2x'} + \boxed{x^3} = t\cos t$

$$y(t) = \frac{x(t)^3}{3} \to y' = x^2\cdot x' : \quad ty' + 3y = t\cos t \qquad \text{(ЛНУ)}$$

б) $\boxed{x'\cos x} = \boxed{\frac{\sin x}{t}} - \boxed{\sin^2 x}$

$\boxed{(\cos^2 x)' = 2\cos x\cdot(-\sin x)}$

$$y(t) = \sin x(t) \to y' = \cos x\cdot x' : \quad y' = \frac{y}{t} - y^2 \qquad \text{(РИК, БЕР } \alpha = 2)$$

в) $\boxed{x'\mathrm{tg}\,x} + 4t^3\boxed{\cos^3 x} = 2t$

$$x'\mathrm{tg}\,x = \frac{x'\sin x}{\boxed{\cos x}}, \quad y(t) = \cos x(t) \to y' = -\sin x\cdot x' : \quad \frac{-y'}{y} + 4t^3y^3 = 2t, \quad y' - 4t^3y^4 = -2ty \qquad \text{(БЕР, } \alpha = 4)$$

г) $t\boxed{e^x x'} - 2t\boxed{e^{x/2}} = 4\boxed{e^x}$

$$y(t) = e^{x(t)} \to y' = e^x\cdot x' : \quad ty' - 2t\sqrt{y} = 4y \qquad (\text{БЕР } \alpha = \tfrac{1}{2})$$

$\boxed{3}$ $x(t) \rightsquigarrow y(u)$

(4) $$x' = \frac{x\left((\ln x)^2 + t\right)}{2t^{3/2}}, \quad t > 0,\ x > 0$$

Решение уравнения: $u = \sqrt{t} \Rightarrow t = u^2$

$y = \ln x \Rightarrow x = e^y$

$$\frac{dx}{dt} \rightsquigarrow \frac{dy}{du}$$

$$\frac{dx}{dt} = \boxed{\frac{dx}{\ }}\boxed{\frac{dy}{\ }}\cdot\boxed{\frac{du}{dt}} = \frac{dy}{\ }\cdot\frac{e^y}{\ }$$

$dt \quad du$

$$\frac{dx}{dt} = \boxed{\frac{dx}{dy}} \cdot \boxed{\frac{dy}{du}} \cdot \boxed{\frac{du}{dt}} = \frac{dy}{du} \cdot \frac{e^y}{2u}$$

$$\frac{dx}{dy} = \frac{d(e^y)}{dy} = e^y \qquad \frac{du}{dt} = \frac{d(\sqrt{t})}{dt} = \frac{1}{2\sqrt{t}} = \frac{1}{2u}$$

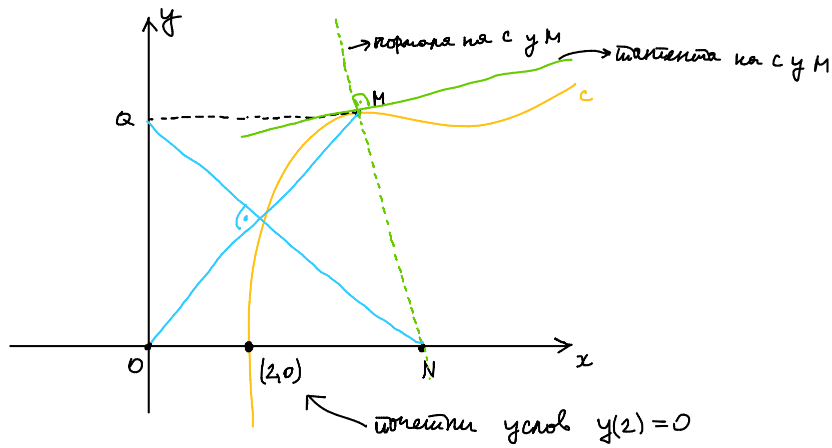
$$\frac{dy}{du} \cdot \frac{e^y}{\boxed{2u}} = \frac{e^y\,(y^2+u^2)}{\boxed{2}u^{\boxed{3}\,2}} \Rightarrow \frac{dy}{du} = \frac{y^2+u^2}{u^2} = \left(\frac{y}{u}\right)^2 + 1 \quad (\text{ХОМ})$$

$\vdots$

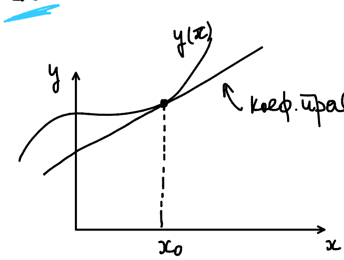
$$y(u) \rightsquigarrow x(t) = e^{y(t)} = e^{y(u^2)}$$

(5) Свака тачка $M$ криве $c$ задовољава следеће: $Q$ - пројекција $M$ на $y$-осу, нормала на $c$ у $M$ сече $x$-осу у $N$, $O$ коорд. почетак. $\forall M \in c$, $QN \perp OM$ и $c$ пролази кроз $(2,0)$, нађи $c$.

$c: y(x) = ?$




Где је ДЈ?




коеф. правца: $k = y'(x_0) = \left.\frac{dy}{dx}\right|_{x=x_0}$

$M(x_0, y(x_0))$

$Q = \pi_y M, \quad \left.\begin{array}{l} x_Q = 0 \\ y_Q = y_M \end{array}\right\} \quad Q(0, y(x_0))$

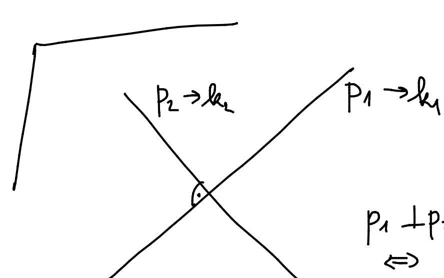
$O(0,0)$

$N(x_N, 0)$

$y_N = 0 \quad (N \in x\text{-оси})$

$x_N = ?$

$Z(x_z, y_z)$



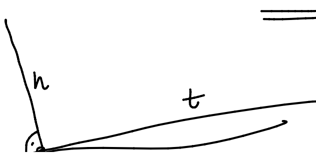

$p_1 \perp p_2$

$\Leftrightarrow$

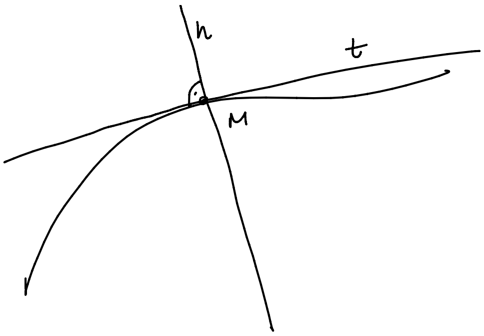
$k_1 \cdot k_2 = -1$

$\Leftrightarrow$

$k_2 = -\frac{1}{k_1}$




$k_t = y'(x$




$k_2 = -\frac{1}{k_1}$

$k_t = y'(x_0)$

$k_n = -\frac{1}{y'(x_0)}$

$M, N \in n$

$\Rightarrow k_n = \frac{y_N - y_M}{x_N - x_M} = \frac{0 - y(x_0)}{x_N - x_0}$

$\frac{1}{y'(x_0)} = \frac{y(x_0)}{x_N - x_0} \Rightarrow x_N = x_0 + y(x_0)y'(x_0)$

$N(x_0 + y(x_0)y'(x_0), 0)$

$\underline{\underline{OM \perp NQ}}$:

$OM = p_1$: $k_1 = \frac{y_M - y_0}{x_M - x_0} = \frac{y_M}{x_M} = \frac{y(x_0)}{x_0}$

$NQ = p_2$: $k_2 = \frac{y_Q - y_N}{x_Q - x_N} = \frac{y(x_0) - 0}{0 - x_0 - y(x_0)y'(x_0)}$

$p_1 \perp p_2 \Leftrightarrow k_1 \cdot k_2 = -1$

$\frac{y(x_0)}{x_0} \cdot \frac{y(x_0)}{-(x_0 + y(x_0)y'(x_0))} = -1 \Rightarrow y^2(x_0) = x_0^2 + x_0 y(x_0) y'(x_0) \to$ ДЈ!

$\forall x_0$ (локално)

ДЈ: $x_0$ - променљива $x_0 \to x$

$y$ - непозната фја $y(x_0) \to y$

$y'(x_0) \to y'$

$\underline{y^2} = x^2 + x\underline{yy'}$, $y(2) = 0$

смена: $\left.\begin{array}{l} z = y^2 \\ z' = 2yy' \end{array}\right\}$ $z = x^2 + x \cdot \frac{z'}{2} \to$ лин

$z(2) = y(2)^2 = 0^2 = 0.$