$\boxed{6}$ Рикатијева дј.

$$x' = p(t)x^2 + q(t)x + r(t) \qquad p, q, r : (a, b) \to \mathbb{R} \text{ непр.}$$

$p \equiv 0$: лин

$r \equiv 0$: БЕР $(\alpha = 2)$

смена: $x_p(t)$ – партикуларно решење

$$x(t) = x_p(t) + \frac{1}{y(t)} \qquad x(t) \rightsquigarrow y(t)$$

↳ лин

$$x' = x_p' + \left(\frac{1}{y}\right)' = x_p' - \frac{1}{y^2}y'$$

x_p: $x_p' = p x_p^2 + q x_p + r$

$$x_p' - \frac{y'}{y^2} = p\left(x_p + \frac{1}{y}\right)^2 + q\left(x_p + \frac{1}{y}\right) + r$$

$$\underbrace{x_p'}_{=} - \frac{y'}{y^2} = \underbrace{\left(p x_p^2 + q x_p + r\right)}_{=} + \frac{2 p x_p}{y} + \frac{p}{y^2} + \frac{q}{y} \Big/ \cdot y^2$$

$$-y' = 2 p x_p y + p + q y \Rightarrow y' + y(2 p x_p + q) = -p$$

ОР: $\begin{cases} x(t) \\ x_p(t) \end{cases}$

(1) а) $\overset{n+1}{t(2t-1)x'} + \overset{2n}{x^2} - \overset{n+1}{(4t+1)x} + \overset{n}{4t} = 0$

б) $x' = \dfrac{2\sin t - x^2 \sin t \cos^2 t}{\cos^2 t}$

в) $x' + x^2 + \dfrac{4x}{t} + \dfrac{2}{t^2} = 0$

а) $x_p = ?$

↳ у облику полинома

$x_p = at + b$

$x_p' = a$

$$t(2t-1)\cdot a + (at+b)^2 - (4t+1)(at+b) + 4t = 0$$

$$t^2\underbrace{\left(2a + a^2 - 4a\right)}_{=0} + t\underbrace{\left(-a + 2ab - 4b - a + 4\right)}_{=0} + \underbrace{\left(b^2 - b\right)}_{=0} = 0$$

$a^2 - 2a = 0 \longrightarrow a \in \{0, 2\}$

$2ab - 2a - 4b + 4 = 0$

$b^2 - b = 0 \longrightarrow b \in \{0, 1\}$

$(a, b) \in \{\cancel{(0,0)}, (0,1), (2,0), (2,1)\}$ ✓ ✓ ✓

$x_p = 1$

$x_p = 2t$

$x_p = 2t+1$

$$(a,b)\in\{\cancel{(0,0)},(0,1),(2,0),(2,1)\}$$

$x_p=1$

$x_p=2t$

$x_p=2t+1$

$x_p=1$: $\quad x=\overset{2t}{1}+\frac{1}{y}$

$$x'=-\frac{y'}{y^2}$$

$$t(2t-1)\left(-\frac{y'}{y^2}\right)+\left(1+\frac{1}{y}\right)^2-(4t+1)\left(1+\frac{1}{y}\right)+4t=0$$

$$t(2t-1)\left(-\frac{y'}{y^2}\right)+1+\frac{2}{y}+\frac{1}{y^2}-(4t+1)-\frac{4t+1}{y}+4t=0\ /\cdot y^2$$

$$-y't(2t-1)+2y+1-(4t+1)y=0$$

$$y'+\frac{4t-1}{t(2t-1)}y=\frac{1}{t(2t-1)}\to \text{ЛНЈ} \quad \therefore \qquad y(t) \rightsquigarrow x(t)$$

хинтови: б) $x'=2\frac{\sin t}{\cos^2 t}-x^2\sin t$

$$\left(\frac{1}{\cos t}\right)'=-\frac{1}{\cos^2 t}\cdot(-\sin t)=\frac{\sin t}{\cos^2 t}$$

$$x_p=\frac{a}{\cos t} \quad \ddots$$

в) $x'+x^2+\frac{4x}{t}+\frac{2}{t^2}=0$

$$\left(\frac{1}{t}\right)'=-\frac{1}{t^2}$$

$$x_p=\frac{a}{t} \quad \ddots$$

Смене

$\boxed{1}$ $x(t)\rightsquigarrow x(u)$

(2) $2t\,x''(t)+(1+\sqrt{t})\,x'(t)-x(t)=t+\sin(\sqrt{t}), \quad t>0$

Доказати да се сменом $u=\sqrt{t}$ своди на $x''+x'-2x=2u^2+2\sin u$.

$$\frac{df}{dt}=\frac{du}{dt}\cdot\frac{df}{du}$$

$x(t)\to x(u)$

$$\frac{dx}{dt}\rightsquigarrow\frac{dx}{du} \qquad \frac{dx}{dt}=\frac{dx}{du}\cdot\frac{du}{dt}=x'_u\cdot\frac{1}{2u}$$

$$\frac{d^2x}{dt^2}\rightsquigarrow\frac{d^2x}{du^2}$$

$$\frac{d}{dt}(x'_u)\cdot\frac{1}{2u}+x'_u\cdot\frac{d}{dt}\left(\frac{1}{2u}\right)=\ldots$$

$$\frac{d^2x}{dt^2}=\frac{d}{dt}\left(\frac{dx}{dt}\right)=\frac{d}{dt}\left(x'_u\cdot\frac{1}{2u}\right)=$$

$u=\sqrt{t}$

$t=u^2$

$$=\frac{du}{dt}\cdot\frac{d}{du}\left(x'_u\cdot\frac{1}{2u}\right)=\frac{1}{2u}\cdot\left(\frac{d}{du}(x'_u)\cdot\frac{1}{2u}+x'_u\cdot\frac{d}{du}\left(\frac{1}{2u}\right)\right)=$$

$$\frac{du}{dt}=\frac{d(\sqrt{t})}{dt}=\frac{1}{2\sqrt{t}}=\frac{1}{2u}$$

$$=\frac{1}{2u}\cdot\left(x''_{uu}\cdot\frac{1}{2u}+x'_u\cdot\left(-\frac{1}{2u^2}\right)\right)$$

$$2u^2\left(\frac{x''_{uu}}{4u^2}-\frac{x'_u}{4u^3}\right)+(1+u)\,x'_u\cdot\frac{1}{2u}-x=u^2+\sin u$$

$$\frac{x''_{uu}}{2} - \underbrace{\frac{x'_u}{2u}}_{\times} + \underbrace{\frac{x'_u}{2u}}_{\times} + \frac{x'_u}{2} - x = u^2 + \sin u \quad / \cdot 2$$

$$x''_{uu} + x'_u - 2x = 2u^2 + 2\sin u$$

$\boxed{2}$ $x(t) \rightsquigarrow y(t)$

$y = f(x) \rightsquigarrow y' = f'(x) \cdot x'$

③ а) $t\boxed{x^2x'} + \boxed{x^3} = t\cos t$

$y(t) = x(t)^3 \to y' = 3x^2x' : \quad \frac{y'}{3} \cdot t + y = t\cos t \quad$ (ЛНН)

б) $\boxed{x'\cos x} = \frac{\boxed{\sin x}}{t} - \boxed{\sin^2 x}$

$y(t) = \sin x(t) \to y' = \cos x \cdot x' : \quad y' = \frac{y}{t} - y^2 \quad$ (РИК, БЕР $\alpha = 2$)

в) $\boxed{x'\operatorname{tg} x} + 4t^3\boxed{\cos^3 x} = 2t$

$x'\operatorname{tg} x = x' \cdot \frac{\sin x}{\cos x}$, $\quad y(t) = \cos x(t) \to y' = -\sin x \cdot x' : \quad \frac{-y'}{y} + 4t^3y^3 = 2t \Rightarrow y' - 4t^3y^4 = -2ty$

(БЕР $\alpha = 4$)

г) $t\boxed{e^x x'} - 2t\boxed{e^{x/2}} = 4\boxed{e^x}$

$y(t) = e^{x(t)} \to y' = e^x \cdot x' : \quad ty' - 2t\sqrt{y} = 4y \quad$ (БЕР $\alpha = \frac{1}{2}$)

$(e^{x(t)/2})$

$\boxed{3}$ $x(t) \rightsquigarrow y(u)$

④ $x' = \frac{x((\ln x)^2 + t)}{2t^{3/2}}$, $\quad t > 0, \ x > 0$

$\frac{dx}{dt} \rightsquigarrow \frac{dy}{du}$

Решити увођењем: $u = \sqrt{t} \Rightarrow t = u^2$

$y = \ln x \Rightarrow x = e^y$

$$\frac{dx}{dt} = \boxed{\frac{dx}{dy}} \cdot \boxed{\frac{dy}{du}} \cdot \boxed{\frac{du}{dt}} = \frac{dy}{du} \cdot \frac{e^y}{2u}$$

$$\frac{du}{dt} = \frac{d(\sqrt{t})}{dt} = \frac{1}{2\sqrt{t}} = \frac{1}{2u}$$

$$\frac{dx}{dy} = \frac{d(e^y)}{dy} = e^y$$

$y = \ln x$

$x = e^y$

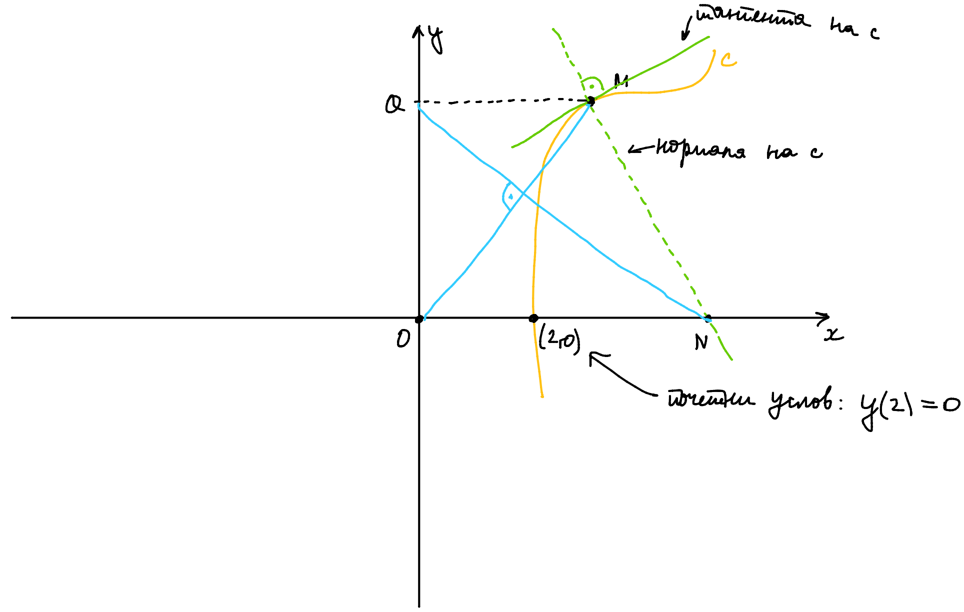
$$\frac{dy}{du} \cdot \frac{e^y}{2u} = \frac{e^y(y^2 + u^2)}{2u^3} \Rightarrow \frac{dy}{du} = \frac{y^2 + u^2}{u^2} = \left(\frac{y}{u}\right)^2 + 1 \quad \text{(ХОМ)}$$

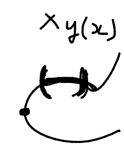
$\vdots$

$y(u) \rightsquigarrow x(t) = e^{y(t)} = e^{y(u^2)}$

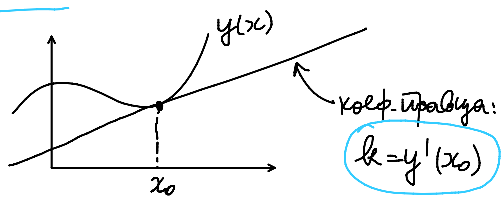
(5) Наћи криву c, ако важи следеће. $M \in c$ се пројектује на y-осу у Q, а нормала на c у M сече x-осу у N. Нека је O коорд. почетак. $\forall M \in c$, $QN \perp OM$ и c пролази кроз $(2,0)$.

$c: y(x) = ?$





Где је ДЈ?



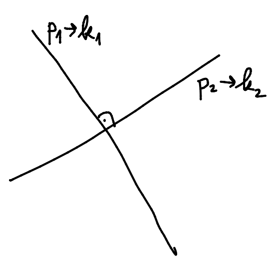
$\boxed{k = y'(x_0)}$

$M(x_0, y(x_0))$

$\left.\begin{array}{l} y_Q = y_M \\ x_Q = 0 \end{array}\right\} Q(0, y(x_0))$

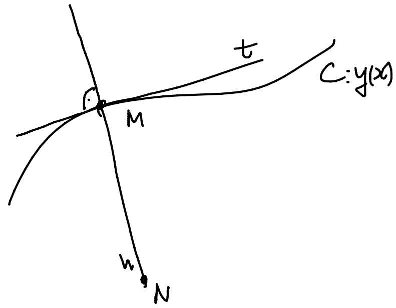
$O(0,0)$

$N(\underset{?}{x_N}, 0)$



$p_1 \perp p_2 \iff k_1 \cdot k_2 = -1$

$k_2 = -\frac{1}{k_1}$



$k_t = y'(x_0)$

$k_n = -\dfrac{1}{y'(x_0)}$

$$k_n = \frac{y_M - y_N}{x_M - x_N} ? = \frac{y(x_0) - 0}{x_0 - x_N}$$

$$-\frac{1}{y'(x_0)} = \frac{y(x_0)}{x_0 - x_N}$$

$$x_N - x_0 = y(x_0) \cdot y'(x_0)$$

$$x_N = x_0 + y(x_0)\, y'(x_0)$$

$OM \perp QN$:

$$OM = p_1: \quad k_1 = \frac{y_M - y_0}{x_M - x_0} = \frac{y(x_0) - 0}{x_0 - 0} = \frac{y(x_0)}{x_0}$$

$$QN = p_2: \quad k_2 = \frac{y_Q - y_N}{x_Q - x_N} = \frac{y(x_0) - 0}{0 - (x_0 + y(x_0)\, y'(x_0))} = -\frac{y(x_0)}{x_0 + y(x_0)\, y'(x_0)}$$

$p_1 \perp p_2$: $k_1 \cdot k_2 = -1$

$$-\frac{y(x_0)}{x_0} \cdot \frac{y(x_0)}{x_0 + y(x_0) \cdot y'(x_0)} = -1$$

$$y^2(x_0) = x_0^2 + x_0 \cdot y(x_0) \cdot y'(x_0) \qquad \forall x_0 \in (a, b)$$

ДУ:
x_0 – променлива $\quad x_0 \to x$
y – непозната $\quad y(x_0) \to y$

$$\boxed{y^2} = x^2 + x\boxed{yy'}, \quad y(2) = 0$$

$$\left.\begin{array}{l} z = y^2 \\ z' = 2yy' \end{array}\right\} \quad \begin{array}{l} z = x^2 + x \cdot \dfrac{z'}{2} \\ z(2) = 0 \end{array} \quad \to \text{лин}$$

$\ddots$