8 - уторак 20.04.2021.

г) $A = \begin{bmatrix} -2 & -5 \\ 2 & 2 \end{bmatrix}$; д) $A = \begin{bmatrix} -1 & -1 \\ 2 & -3 \end{bmatrix}$; ђ) $A = \begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix}$.

г) $A = \begin{bmatrix} -2 & -5 \\ 2 & 2 \end{bmatrix} \rightsquigarrow \lambda_{1/2} = \pm i\sqrt{6}$ – центар $X^* = (0,0)$

$$(A - i\sqrt{6}E)\gamma = 0$$

$$(-2 - i\sqrt{6})a - 5b = 0 \Rightarrow b = -\frac{2+\sqrt{6}i}{5}a$$

.. - - - -

$$\gamma = \begin{bmatrix} 5 \\ -2-i\sqrt{6} \end{bmatrix}$$

$$\psi(t) = e^{i\sqrt{6}t}\cdot\gamma = (\cos(\sqrt{6}t) + i\sin(\sqrt{6}t))\cdot\begin{bmatrix} 5 \\ -2-i\sqrt{6} \end{bmatrix} \begin{matrix} \rightsquigarrow \mathrm{Re}\,\psi \\ \rightsquigarrow \mathrm{Im}\,\psi \end{matrix}$$

$$X(t) = c_1\begin{bmatrix} 5\cos(\sqrt{6}t) \\ -2\cos(\sqrt{6}t) + \sqrt{6}\sin(\sqrt{6}t) \end{bmatrix} + c_2\begin{bmatrix} 5\sin(\sqrt{6}t) \\ -2\sin(\sqrt{6}t) - \sqrt{6}\cos(\sqrt{6}t) \end{bmatrix}$$ → тешко нацртати

$$A = T\cdot\underbrace{\begin{bmatrix} 0 & \sqrt{6} \\ -\sqrt{6} & 0 \end{bmatrix}}_{J}\cdot T^{-1}$$

$$X' = AX \quad / T^{-1}$$

$$Y = T^{-1}X$$
$$Y' = T^{-1}X'$$

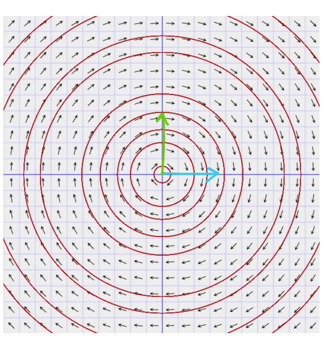
$$Y' = T^{-1}A\cdot X$$

$$Y' = J\underbrace{T^{-1}X} = JY$$ → нацртати

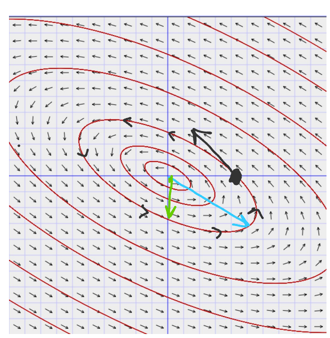
$$J = \begin{bmatrix} 0 & \sqrt{6} \\ -\sqrt{6} & 0 \end{bmatrix}$$ → знамо да су кругови

$$T = [\mathrm{Re}\,\gamma\downarrow \quad \mathrm{Im}\,\gamma\downarrow] = \begin{bmatrix} 5 & 0 \\ -2 & -\sqrt{6} \end{bmatrix}$$

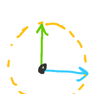
J

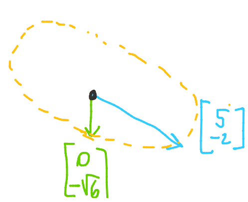


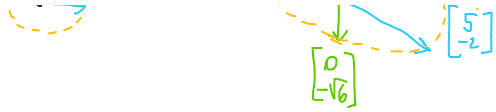
A



$$X = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow X' = \begin{bmatrix} -2 & -5 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$







в) $A = \begin{bmatrix}-1 & -1\\ 2 & -3\end{bmatrix}$

$\lambda_{1/2} = -2 \pm i$

$X^* = (0,0)$

$\alpha \pm i\beta$ $(\alpha, \beta \neq 0)$ – спирала (фокус)

$\alpha > 0$: нестабилна спирала

$\alpha < 0$: стабилна спирала

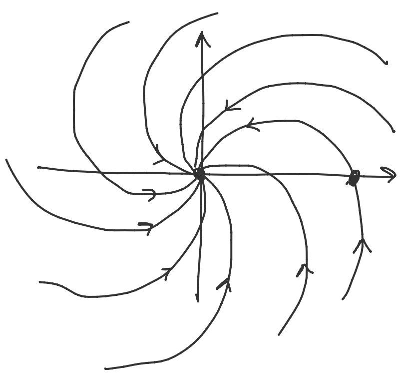
$A = T J T^{-1}$

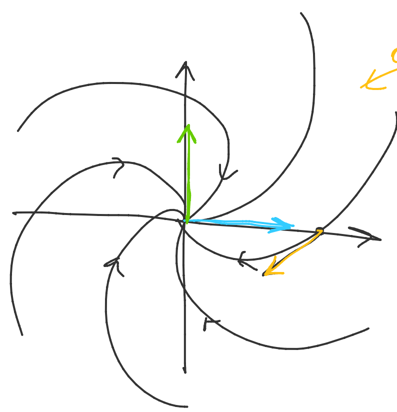
$J = \begin{bmatrix}-2 & 1\\ -1 & -2\end{bmatrix}$

$T = \begin{bmatrix}1 & 0\\ 1 & -1\end{bmatrix}$

$(A - (-2+i)E)v = 0$ ∴ $v = \begin{bmatrix}1\\ 1-i\end{bmatrix}$

J



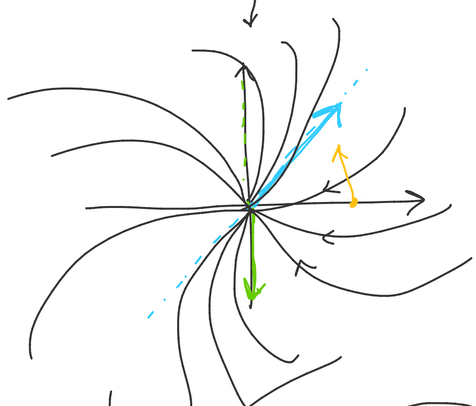


ова јед. $x' = J \cdot x = \begin{bmatrix}-2\\-1\end{bmatrix}$

$x = \begin{bmatrix}1\\0\end{bmatrix}$

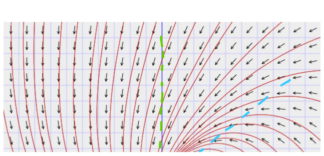
$\alpha < 0 \rightarrow$ иду ФТ ка X^*

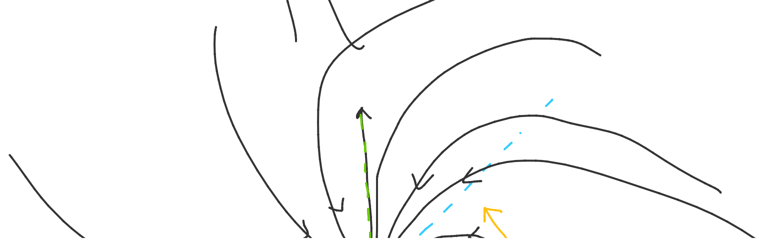
$\alpha > 0 \rightarrow$ иду ФТ од X^*

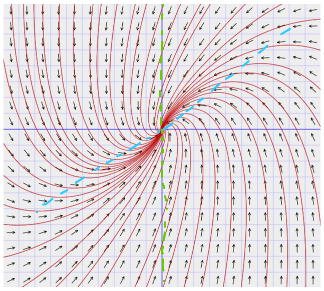


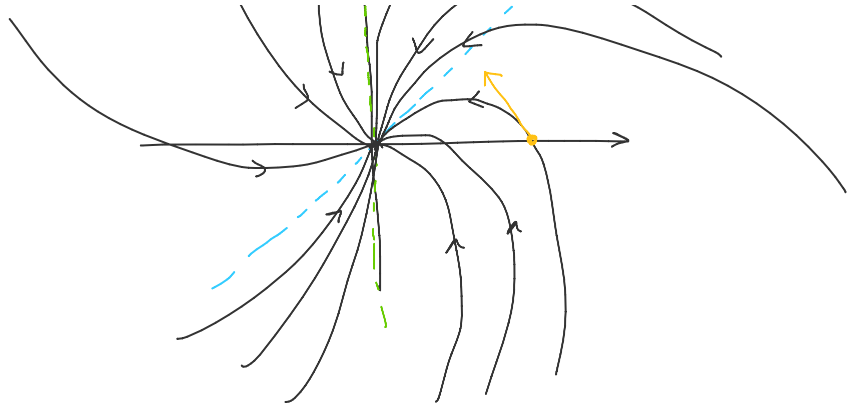
$X = \begin{bmatrix}1\\0\end{bmatrix}$

$X' = AX = \begin{bmatrix}-1\\2\end{bmatrix}$









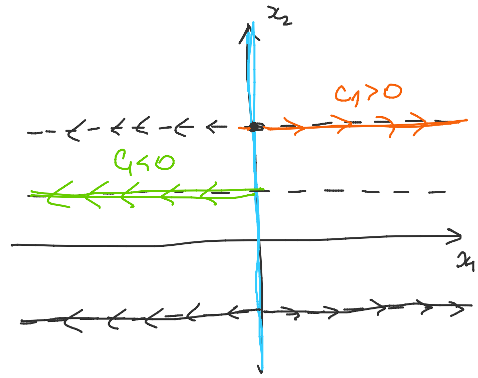
ђ) $A=\begin{bmatrix}5 & 0\\ 0 & 0\end{bmatrix}$

$$\left.\begin{aligned} x_1' &= 5x_1 \\ x_2' &= 0 \end{aligned}\right\} \quad \begin{aligned} x_1 &= c_1 e^{5t} \\ x_2 &= c_2 \end{aligned}$$

$c_2 \in \mathbb{R}$

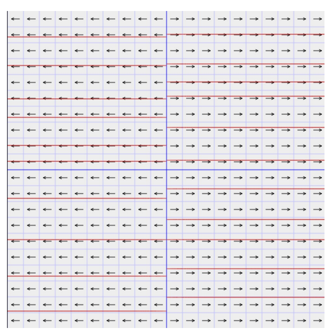
$c_1>0: \ x_1: 0 \to \infty$

$c_1<0: \ x_1: 0 \to -\infty$



$X^* = ?$

$\left.\begin{aligned} x_1 &= 0 \\ 0 &= 0 \end{aligned}\right\} (0,s),\ s\in\mathbb{R} \to$ неизоловани еквилибријуми



ФТ су полуправе које почињу у неком еквилибријуму

1. Скицирати фазни портрет динамичког система $X' = AX$, ако је:

а) $A=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$ б) $A=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$ в) $A=\begin{bmatrix}3 & 1\\ -4 & -1\end{bmatrix}$ г) $A=\begin{bmatrix}-3 & 1\\ -1 & -1\end{bmatrix}$ д) $A=\begin{bmatrix}0 & 2\\ 0 & 0\end{bmatrix}$.

а) $A=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$ $\lambda_1=\lambda_2=2$

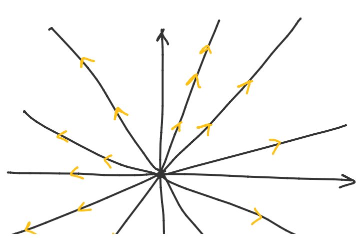
$\hookrightarrow$ нестабилна звезда (сингуларни чвор)

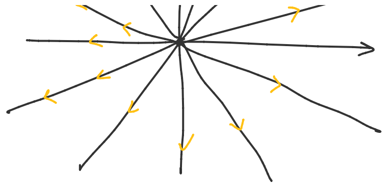
$J=\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}$

$\lambda>0 \to$ нестабилно

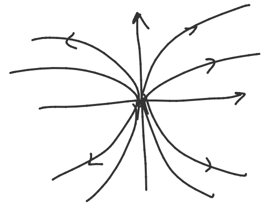
$\lambda<0 \to$ стабилно

$X^* = (0,0)$

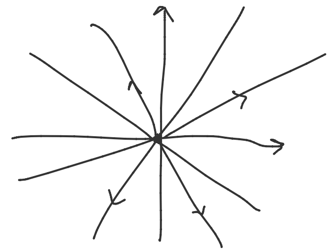




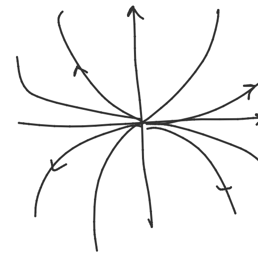
$\lambda_2>\lambda_1>0$



$\lambda_2=\lambda_1>0$



$\lambda_1>\lambda_2>0$



в) $A=\begin{bmatrix}3&1\\-4&-1\end{bmatrix}$ $\qquad \lambda_1=\lambda_2=1$

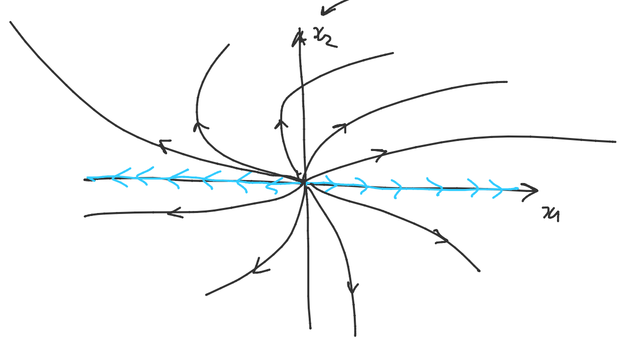
$X^*=(0,0)$ $\qquad (A-E)\nu=0 \rightsquigarrow \nu=\begin{bmatrix}1\\-2\end{bmatrix}\to$ један Жорданов блок $\qquad J=\begin{bmatrix}1&1\\0&1\end{bmatrix}$

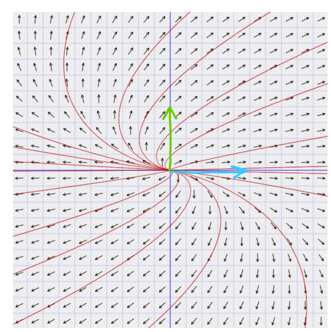
нестабилан дегенерисан чвор

$\lambda=1>0$

$x_1'=x_1+x_2$
$x_2'=x_2$

$x_2\equiv 0,\ x_1=c_1e^t$

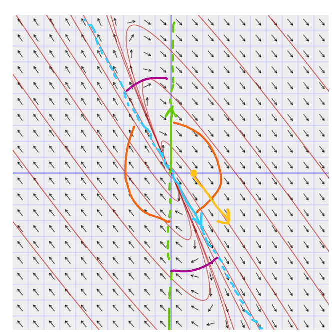




$(A-E)\nu_1=\nu \rightsquigarrow \nu_1=\begin{bmatrix}0\\1\end{bmatrix}$ (уопштени за ν)

$T=\begin{bmatrix}1&0\\-2&1\end{bmatrix}$

орбите
решење



$X=\begin{bmatrix}1\\0\end{bmatrix},\ X'=AX=\begin{bmatrix}3\\-4\end{bmatrix}$

2. Нека су $a,b\in\mathbb{R}$ и нека је $a\neq\pm b$. Свести диференцијалну једначину $y''+2ay'+b^2y=0$ на систем диференцијалних једначина. У зависности од параметара a и b испитати тип еквилибријума и скицирати фазне портрете.

$x_1=y$ $\qquad x_1'=x_2$

ренцијалних једначина. У зависности од параметара a и b испитати тип еквилибријума и скицирати фазне портрете.

$$\left.\begin{aligned} x_1 &= y \\ x_2 &= y' \end{aligned}\right\} \qquad \left.\begin{aligned} x_1' &= x_2 \\ x_2' &= -b^2 x_1 - 2a x_2 \end{aligned}\right\}$$

$$\Downarrow$$

$$x_2' = y'' = -2a\underbrace{y'}_{x_2} - b^2 \underbrace{y}_{x_1}$$

$$A = \begin{bmatrix} 0 & 1 \\ -b^2 & -2a \end{bmatrix}, \quad \det(A - \lambda E) = 0$$

$$(0-\lambda)(-2a-\lambda) - 1\cdot(-b^2) = 0$$

$$\lambda^2 + 2a\lambda + b^2 = 0$$

$$D = (2a)^2 - 4b^2 = 4(a^2-b^2) \neq 0 \rightarrow a \neq \pm b$$

1° $a^2-b^2>0$, $D>0$, $\lambda_{1/2} = \dfrac{-2a \pm \sqrt{4(a^2-b^2)}}{2} = -a \pm \sqrt{a^2-b^2}$ $\quad (\lambda_1 \neq \lambda_2)$

2° $a^2-b^2<0$, $D<0$, $\lambda_{1/2} = \dfrac{-2a \pm i\sqrt{4(b^2-a^2)}}{2} = -a \pm i\sqrt{b^2-a^2}$

$$X^* = ? \qquad AX = 0 \Rightarrow \left.\begin{aligned} x_2 &= 0 \\ -b^2 x_1 - 2a x_2 &= 0 \end{aligned}\right\} \begin{aligned} x_2 &= 0 \\ b^2 x_1 &= 0 \end{aligned}$$

1° $a^2-b^2>0$ $\quad$ ($a=0$ немогуће)

1.1° $b=0$ $\quad \{(\lambda,0) \mid \lambda \in \mathbb{R}\} \ni X^* \Rightarrow$ неизоловани еквилибријуми

1.2° $b \neq 0$ $\quad \overset{\lambda_1 \shortparallel}{-a+\sqrt{a^2-b^2}} > \overset{\shortparallel \lambda_2}{-a-\sqrt{a^2-b^2}}$

у осталим случајевима имамо само $X^* = (0,0)$ као еквилибријум

$1.2.1^\circ$ $a>0$, $\lambda_2<0$ свакако

$\lambda_1 < 0 \Leftrightarrow -a+\sqrt{a^2-b^2}<0$

$\sqrt{a^2-b^2} < a \; /^2$ (обе >0)

$a^2-b^2 < a^2$ ✓

$\lambda_2 < \lambda_1 < 0 \rightarrow$ стабилан чвор

$1.2.2^\circ$ $a<0$, $\lambda_1>0$

$\lambda_2 > 0 \Leftrightarrow -a > \sqrt{a^2-b^2} \; /^2$ (обе >0)

$a^2 > a^2-b^2$ ✓

$0 < \lambda_2 < \lambda_1 \rightarrow$ нестабилан чвор

2° $a^2-b^2<0$

$2.1^\circ \quad a=0 \qquad \lambda_{1/2} = \pm i\sqrt{b^2-a^2} \to$ центар

$2.2^\circ \quad a>0 \qquad Re(\lambda_1) = Re(\lambda_2) < 0$, $Im(\lambda_1) \neq 0 \to$ стабилна спирала

$2.3^\circ \quad a<0 \qquad Re(\lambda_1) = Re(\lambda_2) > 0$, $Im(\lambda_1) \neq 0 \to$ нестабилна спирала