домаћи: $x_1' = x_2 - x_1 x_2^2$

$x_2' = -x_1^3$

а) $V(x_1,x_2) = ax_1^2 + bx_2^2$ се не може користити као фја Љапунова

б) $V(x_1,x_2) = ax_1^4 + bx_2^2$ ← тражити у овом облику

Системи у симетричном облику

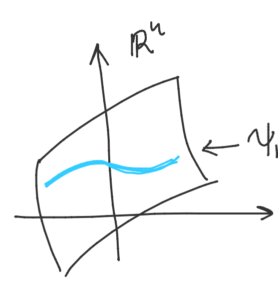
$$\frac{dx_1}{f_1(x_1,\dots,x_n)} = \frac{dx_2}{f_2(x_1,\dots,x_n)} = \dots = \frac{dx_n}{f_n(x_1,\dots,x_n)}$$ ← систем у симетричном облику

$$\Updownarrow$$

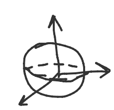
$$\left.\begin{aligned} x_2' &= \frac{dx_2}{dx_1} = \frac{f_2(x_1,\dots,x_n)}{f_1(x_1,\dots,x_n)} \\ x_3' &= \frac{dx_3}{dx_1} = \frac{f_3(\dots)}{f_1(\dots)} \\ &\vdots \\ x_n' &= \frac{dx_n}{dx_1} = \frac{f_n}{f_1} \end{aligned}\right\}$$ ← систем у нормалном облику

- Изабрали смо x_1 за независну променљиву $\rightsquigarrow$ $f_1 \neq 0$ захтевамо
- Могли смо да изаберемо и другу нез. пр.
- Први интеграл (ψ_k) је фја која је константна дуж решења

$$\psi_1(x_1,\dots,x_n) = c_1$$
$$\psi_2(x_1,\dots,x_n) = c_2$$
$$\vdots$$
$$\psi_{n-1}(x_1,\dots,x_n) = c_{n-1}$$
$$c_1,\dots,c_{n-1} \in \mathbb{R}$$



← $\psi_k = c_k$ хиперповрш у $\mathbb{R}^n$



$\psi(x,y,z) = x^2+y^2+z^2$

$\psi = c > 0 \to$ сфера

- Први интеграли $\psi_1,\dots,\psi_{n-1}$ су лин. нез акко:

$$\frac{D(\psi_1,\dots,\psi_{n-1})}{D(x_2,\dots,x_n)} \neq 0$$

↳ изоставимо нез. пром.

• Опште решење: $(\psi_1, \dots, \psi_{n-1}) = const \in \mathbb{R}^{n-1}$ (у имплицитном облику)

① Решити системе у симетричном облику:

а) $\dfrac{dx}{x+y^2+z^2} = \dfrac{dy}{y} = \dfrac{dz}{z}$

в) $\dfrac{dx}{4y-3z} = \dfrac{dy}{4x-2z} = \dfrac{dz}{2y-3x}$

б) $\dfrac{dx}{y-u} = \dfrac{dy}{z-x} = \dfrac{dz}{u-y} = \dfrac{du}{x-z}$

г) $\dfrac{dx}{x^2-y^2-z^2} = \dfrac{dy}{2xy} = \dfrac{dz}{2xz}$

а) $3 \to x, y, z \to \psi_1, \psi_2$?

$$y' = \frac{dy}{dz} = \frac{y}{z} \Rightarrow \frac{dy}{y} = \frac{dz}{z} \quad /\int$$

z за независну

$z \neq 0$

$\dots = \dfrac{dz}{z}$

$$\ln|y| = \ln|z| + c_1 \quad / e^{\square}$$

$\vdots$

$$y = c \cdot z \Rightarrow c = \frac{y}{z} \Rightarrow \psi_1(x,y,z) = \frac{y}{z}$$

$c \in \mathbb{R}$

$c = const$

$$x' = \frac{dx}{dz} = \frac{x+y^2+z^2}{z} = \frac{x+(c\cdot z)^2+z^2}{z} = \frac{x}{z} + (c^2+1)\cdot z$$ линеарна x од z

$$x' - \frac{x}{z} = (c^2+1)\cdot z$$

$p(z) = -\dfrac{1}{z}$

$q(z) = (c^2+1)\cdot z$ ...

$$x(z) = c_2 z + (1+c^2)\cdot z^2 = c_2 z + \left(1 + \frac{y^2}{z^2}\right)\cdot z^2 =$$

$$= c_2 z + y^2 + z^2$$

$$\Rightarrow c_2 = \frac{x-y^2-z^2}{z} = \psi_2(x,y,z)$$

Да ли су ψ_1 и ψ_2 незав?

$$\frac{D(\psi_1,\psi_2)}{D(x,y)} = \begin{vmatrix} \frac{\partial\psi_1}{\partial x} & \frac{\partial\psi_1}{\partial y} \\ \frac{\partial\psi_2}{\partial x} & \frac{\partial\psi_2}{\partial y} \end{vmatrix} = \begin{vmatrix} 0 & \frac{1}{z} \\ \frac{1}{z} & -\frac{2y}{z} \end{vmatrix} = -\frac{1}{z^2} \neq 0$$

$$D(x,y) \quad \begin{vmatrix} \frac{\partial \psi_2}{\partial x} & \frac{\partial \psi_2}{\partial y} \end{vmatrix} = \begin{vmatrix} \frac{1}{z} & -\frac{2y}{z} \end{vmatrix} = -\frac{1}{z^2} \neq 0$$

$\Rightarrow \psi_1$ и ψ_2 нез. $\Rightarrow$ ОР: $\left.\begin{array}{l} \psi_1 = C_1 \\ \psi_2 = C_2 \end{array}\right\}$

$C_1, C_2 \in \mathbb{R}$

б) $$\frac{dx}{y-u} = \frac{dy}{z-x} = \frac{dz}{u-y} = \frac{du}{x-z} \rightsquigarrow \psi_1, \psi_2, \psi_3$$

$$\int dA = A + C$$

$$\frac{dx}{y-u} = \frac{dz}{u-y} \quad /\cdot(u-y)$$

$$-dx = dz \quad /\int$$

$$-x = z + C_1$$

$$C_1 = -x - z = \psi_1(x,y,z,u)$$

$$\frac{dy}{z-x} = \frac{du}{x-z} \quad /\cdot(x-z)$$

$$-dy = du \quad /\int$$

$$-y + C_2 = u$$

$$C_2 = u + y = \psi_2(x,y,z,u)$$

$\psi_3 = x+z+y+u = -C_1 + C_2 = const$

ψ_3 је зависан са ψ_1, ψ_2

$$\frac{D(\psi_1, \psi_2, \psi_3)}{D(x,y,z)} = 0$$

$(1)+(3) = (2)+(4)$:

$$\frac{dx}{y-u} + \frac{dz}{u-y} = \frac{dy}{z-x} + \frac{du}{x-z}$$

$$\frac{dx - dz}{y-u} = \frac{du - dy}{x-z} \Rightarrow \frac{d(x-z)}{y-u} = \frac{d(u-y)}{x-z}$$

$$d(f \pm g) = df \pm dg$$

$$(x-z)\,d(x-z) = (y-u)\,d(u-y)$$

$$\underbrace{\overset{A}{(x-z)}\,\overset{dA}{d(x-z)}} + (u-y)\,d(u-y) = 0 \quad /\int$$

$$\int A\,dA = \frac{A^2}{2} + C$$

$$\frac{(x-z)^2}{2} + \frac{(u-y)^2}{2} = C_3 \quad /\cdot 2$$

$$(x-z)^2 + (u-y)^2 = 2C_3 = \tilde{C}_3 = const = \psi_3(x,y,z,u)$$

Нез. пром.? (било шта) Нпр. y : $z - x \neq 0$

$$\frac{D(\psi_1,\psi_2,\psi_3)}{D(x,z,u)} = \begin{vmatrix} -1 & -1 & 0 \\ 0 & 0 & 1 \\ 2(x-z) & 2(z-x) & 2(u-y) \end{vmatrix} = -2(x-z)+2(z-x) = -4(x-z) \neq 0$$

$\Rightarrow$ niez $\Rightarrow$ OP: $(\psi_1,\psi_2,\psi_3) = c \in \mathbb{R}^3$.

B) $\frac{dx}{4y-3z} = \frac{dy}{4x-2z} = \frac{dz}{2y-3x} \rightsquigarrow \psi_1, \psi_2$

uwaga: $\frac{dx}{f_1} = \frac{dy}{f_2} = \frac{dz}{f_3}$

$\rightarrow \frac{\alpha dx + \beta dy}{\alpha f_1 + \beta f_2} = \frac{dz}{f_3}$

$\alpha, \beta = ?$

$$\frac{\alpha dx + \beta dy}{\alpha(4y-3z)+\beta(4x-2z)} = \frac{dz}{2y-3x}$$

$$\frac{\alpha dx + \beta dy}{x(4\beta)+y(4\alpha)+z\underbrace{\boxed{(-3\alpha-2\beta)}}_{=0}} = \frac{dz}{2y-3x}$$

z?

$-3\alpha - 2\beta = 0$

$\beta = 3$

$\alpha = -2$

$$\frac{-2dx+3dy}{\cancel{12x-8y}} = \frac{dz}{\cancel{2y-3x}} \Rightarrow \frac{-2dx+3dy}{-4} = dz \;/\cdot(-4)$$

-4

$-2dx+3dy+4dz = 0 \;/\int$

$-2x+3y+4z = C_1$

$\Rightarrow \psi_1(x,y,z) = -2x+3y+4z$

$$\frac{\alpha x dx + \beta y dy}{\alpha x(4y-3z)+\beta y(4x-2z)} = \frac{z dz}{z(2y-3x)}$$

wyr xy: $4\alpha + 4\beta = 0$

$\alpha = 1, \beta = -1$

$$\frac{x dx - y dy}{-3xz+2yz} = \frac{z dz}{-3xz+2yz} \Rightarrow x dx - y dy = z dz \;/\int$$

$\frac{x^2}{2} - \frac{y^2}{2} = \frac{z^2}{2} + C_2 \;/\cdot 2$

$x^2 - y^2 - z^2 = 2C_2 = \psi_2(x,y,z)$

итр. x нез:

$4y-3z\neq 0$

$$\frac{D(\psi_1,\psi_2)}{D(y,z)} = \begin{vmatrix} 3 & 4 \\ -2y & -2z \end{vmatrix} = -6z+8y = 2(4y-3z)\neq 0$$

$$\Rightarrow \text{ОР:}\quad \left.\begin{array}{l}\psi_1=C_1\\ \psi_2=C_2\end{array}\right\}\quad C_1,C_2\in\mathbb{R}$$

г) $$\frac{dx}{x^2-y^2-z^2} = \underbrace{\frac{dy}{2xy} = \frac{dz}{2xz}}$$

$$\hookrightarrow \frac{dy}{2xy} = \frac{dz}{2xz} \Rightarrow \frac{dy}{y} = \frac{dz}{z} \Rightarrow \dots\ y=c\cdot z \qquad \psi_1 = \frac{y}{z}$$

z нез. $\Rightarrow 2xz\neq 0$

$$x' = \frac{dx}{dz} = \frac{x^2-y^2-z^2}{2xz} = \frac{x}{2z} - \frac{z^2(c^2+1)}{2xz} = \frac{x}{2z} - \frac{z(c^2+1)}{2x} \qquad \frac{1}{x} = x^{-1}$$

Бернулијева: $x' = p(z)x + q(z)x^{\alpha}$

$$p(z) = \frac{1}{2z}$$

$$q(z) = -\frac{z(c^2+1)}{2}$$

$$\alpha = -1$$

смена: $u = x^{1-\alpha} = x^{1-(-1)} = x^2 \dots$

домаћи: решити Бер. јед, $\psi_2 = \dfrac{x^2+y^2+z^2}{z}$, проверити нез. ψ_1 и ψ_2

② Решити систем

$$\left.\begin{array}{l} \dfrac{dy}{dx} = y' = \dfrac{y(x^2+z^2)}{x(z^2-y^2)} \\[2ex] \dfrac{dz}{dx} = z' = \dfrac{z(x^2+y^2)}{x(y^2-z^2)} \end{array}\right\}$$

идеја: представити у сим. облик

$$\frac{dx}{x(z^2-y^2)} = \frac{dy}{y(x^2+z^2)} = \frac{dz}{-z(x^2+y^2)}$$

$$\frac{\alpha x\,dx + \beta y\,dy}{\alpha x^2(z^2-y^2) + \beta y^2(x^2+z^2)} = \frac{z\,dz}{-z^2(x^2+y^2)}$$

$\rightarrow$ нема x^2y^2

$$\frac{}{\alpha x^2(z^2-y^2)+\beta y^2(x^2+z^2)} = \frac{}{-z^2(x^2+y^2)} \rightsquigarrow \text{нема } x^2y^2$$

уз x^2y^2: $-\alpha+\beta=0$

$\alpha=\beta=1$

$$\frac{x\,dx+y\,dy}{x^2z^2+\cancel{y^2z^2}} = \frac{z\,dz}{-z^2(\cancel{x^2+y^2})} \Rightarrow x\,dx+y\,dy=-z\,dz \;/\int$$

$$\frac{x^2}{2}+\frac{y^2}{2}=-\frac{z^2}{2}+c_1$$

$$\psi_1(x,y,z)=\frac{1}{2}(x^2+y^2+z^2)$$

Никако:

$x\,dy+dz=0 \;/\int$

$xy+z=0$

$$\frac{\frac{\alpha}{x}dx+\frac{\beta}{y}dy}{\alpha(z^2-y^2)+\beta(x^2+z^2)} = \frac{\frac{dz}{z}}{-(x^2+y^2)} \leftarrow \text{нема } z^2$$

$$\int\frac{dA}{A}=\ln|A|+c$$

уз z^2: $\alpha+\beta=0$

$\alpha=1,\ \beta=-1$

$$\frac{\frac{dx}{x}-\frac{dy}{y}}{-y^2\cancel{-x^2}} = \frac{\frac{dz}{z}}{-\cancel{(x^2+y^2)}} \Rightarrow \frac{dx}{x}-\frac{dy}{y}-\frac{dz}{z}=0 \;/\int$$

$$\Rightarrow \ln|x|-\ln|y|-\ln|z|=c_2$$

$$\ln\left|\frac{x}{yz}\right|=c_2$$

$\vdots$

$$\frac{x}{yz}=\tilde{c}_2 \overset{\text{инф.}}{\rightsquigarrow} \frac{yz}{x}=\psi_2$$

инф. x нез.:

$\Downarrow$

$x(z^2-y^2)\neq 0$

$$\frac{D(\psi_1,\psi_2)}{D(y,z)} = \begin{vmatrix} y & z \\ \frac{z}{x} & \frac{y}{x} \end{vmatrix} = \frac{y^2}{x}-\frac{z^2}{x}=\frac{y^2-z^2}{x}\neq 0$$

$$\Rightarrow \text{ОР}: \left.\begin{aligned}\psi_1&=c_1\\ \psi_2&=c_2\end{aligned}\right\} \quad c_1,c_2\in\mathbb{R}$$