Понедељак 26.10.2020.

$\boxed{5}$ Бернулијева ДЈ

$$x' + p(t)x = q(t)x^{\alpha} \quad , p,q:(a,b)\to\mathbb{R} \text{ непр.} \qquad \alpha\in\mathbb{R}$$

$\alpha=0$: лин

$\alpha=1$: РП

СМЕНА: $y(t) = x(t)^{1-\alpha}$

$y' = (1-\alpha)\cdot x^{-\alpha}\cdot x'$ $\Big\} \rightsquigarrow$ лин.

① а) $x' = \frac{x}{t} - x^2$

б) $x' + \frac{x}{t} = x^2 \frac{\ln t}{t}$

в) $tx' - 2t\sqrt{x} = 4x$

а) $x' \boxed{-\frac{x}{t}} = -x^{2}$ $\quad \alpha=2$

б) $x' + \frac{1}{t}\cdot x = \frac{\ln t}{t}\cdot x^{2}$ $\quad \alpha=2$

$(t>0)$

$\Big\}\ y = x^{1-2} = \frac{1}{x}$

$\vdots$

в) $tx' - 2t\sqrt{x} = 4x \ /:t$

$x' - 2\sqrt{x} = \frac{4}{t}\cdot x$

$x' \boxed{-\frac{4}{t}}\cdot x = \boxed{2}\sqrt{x} = x^{\frac{1}{2}} \Rightarrow \alpha=\frac{1}{2}$ $\quad /:\sqrt{x}$ $\longrightarrow$ делимо са $/:x^{\alpha}$

$p(t) \qquad q(t)$

$y = x^{1-\frac{1}{2}} = x^{\frac{1}{2}} = \sqrt{x}$

$y' = \frac{1}{2\sqrt{x}}\cdot x' = \frac{1}{2}\cdot x^{-1/2}\cdot x'$

$x'\cdot x^{-1/2} - \frac{4}{t}\cdot x^{1/2} = 2$

$2y' - \frac{4}{t}y = 2 \ /:2$

$y' - \frac{2}{t}y = 1 \quad , \ y(t) = e^{-\int p\,dt}\cdot\left(c + \int q\cdot e^{\int p\,dt}dt\right) \ , c\in\mathbb{R}$

$e^{-2\ln|t|} = \left(e^{\ln|t|}\right)^{-2} = |t|^{-2}$

$p(t) = -\frac{2}{t}$

$q(t) = 1$

$\int p\,dt = \int -\frac{2}{t}\,dt = -2\ln|t|$

$\int q\cdot e^{\int p\,dt}dt = \int 1\cdot e^{-2\ln|t|}dt = \int \frac{1}{|t|^2}dt = \int \frac{1}{t^2}dt = -\frac{1}{t}$

$y(t) = e^{2\ln|t|}\cdot\left(c - \frac{1}{t}\right) = t^2\left(c - \frac{1}{t}\right) = ct^2 - t \ , c\in\mathbb{R}$

$$\boxed{x(t) = y(t)^2 = \left(ct^2 - t\right)^2 \ , c\in\mathbb{R}}$$

$\boxed{6}$ Рикатијева ДЈ

$$x' = p(t)x^2 + q(t)\cdot x + r(t) \quad , \ p,q,r:(a,b)\to\mathbb{R} \text{ непр.}$$

$p\equiv 0 \to$ лин.

$r\equiv 0 \to$ БЕР $(\alpha=2)$

СМЕНА: $x_p(t)$ – једно партикуларно решење

$x(t) \to y(t)$: $x(t) = x_p(t) + \frac{1}{y(t)} \rightsquigarrow$ ЛДЈ $\quad x' = x_p' - \frac{y'}{y^2}$ ⋰

(2) а) $t(2t-1)\cdot x' + x^2 - (4t+1)x + 4t = 0$

б) $x' = \frac{2\sin t - x^2 \sin t \cos^2 t}{\cos^2 t}$

в) $x' + x^2 + \frac{4x}{t} + \frac{2}{t^2} = 0$

а) $t(2t-1) \cdot x' = 2$

$(4t+1) \cdot x = 2$

$1 \cdot x^2 = 2$

ако је x_p пол. 1. степ.

$x_p(t) = at + b$

б) $x' = 2\frac{\sin t}{\cos^2 t} - x^2 \cdot \sin t$ $\quad x_p(t) = \frac{a}{\cos t}$ $\quad x_p' = ? \cdot \frac{\sin t}{\cos^2 t}$

в) $x' + \boxed{x^2} + \frac{4x}{t} + \boxed{\frac{2}{t^2}} = 0$

$x = \frac{?}{t}$ $\quad \frac{x}{t} = \frac{?}{t^2}$

$x^2 = \frac{?}{t^2}$ $\quad x' = -\frac{?}{t^2}$

$x_p(t) = \frac{a}{t}$, $a \in \mathbb{R}$

↳ надамо се да може!

$x_p' = -\frac{a}{t^2}$

$$-\frac{a}{t^2} + \left(\frac{a}{t}\right)^2 + 4 \cdot \frac{a}{t^2} + \frac{2}{t^2} = 0 \;/\cdot t^2$$

$$-a + a^2 + 4a + 2 = 0$$

$$a^2 + 3a + 2 = 0$$

$$(a+2)(a+1) = 0 \xrightarrow{\text{нпр.}} a = -1 \rightsquigarrow x_p = -\frac{1}{t}$$

$$x = x_p + \frac{1}{y} = -\frac{1}{t} + \frac{1}{y}$$

$$x' = \frac{1}{t^2} - \frac{y'}{y^2}$$

$$x' + x^2 + \frac{4x}{t} + \frac{2}{t^2} = 0 \rightsquigarrow \frac{1}{t^2} - \frac{y'}{y^2} + \left(-\frac{1}{t} + \frac{1}{y}\right)^2 + \frac{4}{t}\left(-\frac{1}{t} + \frac{1}{y}\right) + \frac{2}{t^2} = 0$$

$$\frac{1}{t^2} - \frac{y'}{y^2} + \frac{1}{t^2} - \frac{2}{ty} + \frac{1}{y^2} - \frac{4}{t^2} + \frac{4}{ty} + \frac{2}{t^2} = 0$$

$$-\frac{y'}{y^2} + \frac{2}{ty} + \frac{1}{y^2} = 0 \;/\cdot y^2$$

$$-y' + \frac{2}{t}\cdot y + 1 = 0$$

$$y' - \frac{2}{t}\cdot y = 1 \rightsquigarrow \text{ЛДЈ.} \qquad y(t) \rightarrow x(t) = -\frac{1}{t} + \frac{1}{y(t)}$$

Линеарне

$\boxed{1}$ (3) $2t x''(t) + (1+\sqrt{t}) x'(t) - x(t) = t + \sin(\sqrt{t})$, $t > 0$

1 (3) $2t x''(t) + (1+\sqrt{t})x'(t) - x(t) = t + \sin(\sqrt{t})$, $t>0$

$$\Big\downarrow\ u = \sqrt{t}$$

ДУ сведи на: $x'' + x' - 2x = 2u^2 + 2\sin u$

$u=\sqrt{t} \Rightarrow \frac{du}{dt} = \frac{1}{2\sqrt{t}} = \frac{1}{2u}$; $t = u^2$

$$\frac{df}{dt} = \frac{du}{dt}\cdot\frac{df}{du}$$

$x(t) \rightsquigarrow x(u)$

$\frac{dx}{dt} \rightsquigarrow \frac{dx}{du}$

$\frac{d^2u}{dt^2} \rightsquigarrow \frac{d^2x}{du^2}$

$$x'(t) = \frac{dx}{dt} = \frac{dx}{du}\cdot\frac{du}{dt} = \frac{1}{2u}\cdot\underbrace{\boxed{\frac{dx}{du}}}_{x'_u}$$

$$x''(t) = \frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d}{dt}\left(\frac{1}{2u}\cdot\frac{dx}{du}\right) = \frac{d}{dt}\left(\frac{1}{2u}\right)\cdot\frac{dx}{du} + \frac{1}{2u}\cdot\frac{d}{dt}\left(\frac{dx}{du}\right)$$

$$= \frac{du}{dt}\cdot\frac{d}{du}\left(\frac{1}{2u}\right)\cdot\frac{dx}{du} + \frac{1}{2u}\cdot\frac{du}{dt}\cdot\frac{d}{du}\left(\frac{dx}{du}\right) =$$

$$= \frac{1}{2u}\cdot\left(-\frac{1}{2u^2}\right)\cdot\frac{dx}{du} + \frac{1}{2u}\cdot\frac{1}{2u}\cdot\frac{d^2x}{du^2}$$

$$= -\frac{1}{4u^3}\cdot\underbrace{\boxed{\frac{dx}{du}}}_{x'_u} + \frac{1}{4u^2}\cdot\underbrace{\boxed{\frac{d^2x}{du^2}}}_{x''_{uu}}$$

$$2t x''(t) + (1+\sqrt{t})x'(t) - x(t) = t + \sin(\sqrt{t})$$

$$\downarrow$$

$$2\cdot u^2\cdot\left(-\frac{1}{4u^3}\cdot x'_u + \frac{1}{4u^2}\cdot x''_{uu}\right) + (1+u)\cdot\frac{1}{2u}\cdot x'_u - x = u^2 + \sin u$$

$$-\frac{1}{2u}\cdot x'_u + \frac{x''_{uu}}{2} + \frac{x'_u}{2u} + \frac{x'_u}{2} - x = u^2 + \sin u \quad /\cdot 2$$

$$x''_{uu} + x'_u - 2x = 2u^2 + 2\sin u \ \checkmark$$

2 $x(t) \rightsquigarrow y(t)$ $\qquad y = f(x) \rightsquigarrow y' = \underline{f'(x)\cdot x'}$

(4) а) $t\,x^2x' + \boxed{x^3} = t\cos t \longrightarrow \underline{y(t) = x(t)^3} \to y' = 3x^2\cdot x' \to t\cdot\frac{y'}{3} + y = t\cos t$ (лин)

б) $x'\cos x = \frac{\sin x}{t} - \sin^2 x \longrightarrow y(t) = \sin x(t) \to y' = \cos x\cdot x' \to y' = \frac{y}{t} - y^{2}$ $\alpha=2$ БЕР. (Рик)

в) $x'\operatorname{tg} x + 4t^3\cos^3 x = 2t \longrightarrow x'\cdot\operatorname{tg} x = x'\cdot\frac{\sin x}{\cos x} \to y(t) = \cos x(t) \to y' = -\sin x\cdot x' \to \frac{-y'}{y} + 4t^3\cdot y^3 = 2t$

$y' - 4t^3y^{4} = -2ty$ $\alpha=4$ (БЕР)

г) $t e^x x' - 2t e^{x/2} = 4e^x \longrightarrow y(t) = e^{x(t)} \to y' = e^x\cdot x'$

$\to t\cdot y' - 2t\sqrt{y} = 4y$ $\alpha=\frac{1}{2}$ (БЕР)

3 $x(t) \rightsquigarrow y(u)$

$x \rightsquigarrow y$
$t \rightsquigarrow u$

5) Решити ДЈ: $\dfrac{dx}{dt} = \dfrac{x((\ln x)^2 + t)}{2t^{3/2}}$, $t>0$, $x>0$

уводети смене $u = \sqrt{t}$, $y = \ln x$

$$\frac{dy}{du} = \boxed{\frac{dy}{dx}} \cdot \frac{dx}{dt} \cdot \boxed{\frac{dt}{du}} = \frac{2u}{e^y} \cdot \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{dy}{du} \cdot \frac{e^y}{2u}$$

$y = \ln x$

$\dfrac{dy}{dx} = \dfrac{1}{x} = e^{-y}$

$x = e^y \Rightarrow \dfrac{1}{x} = e^{-y}$

$t = u^2$

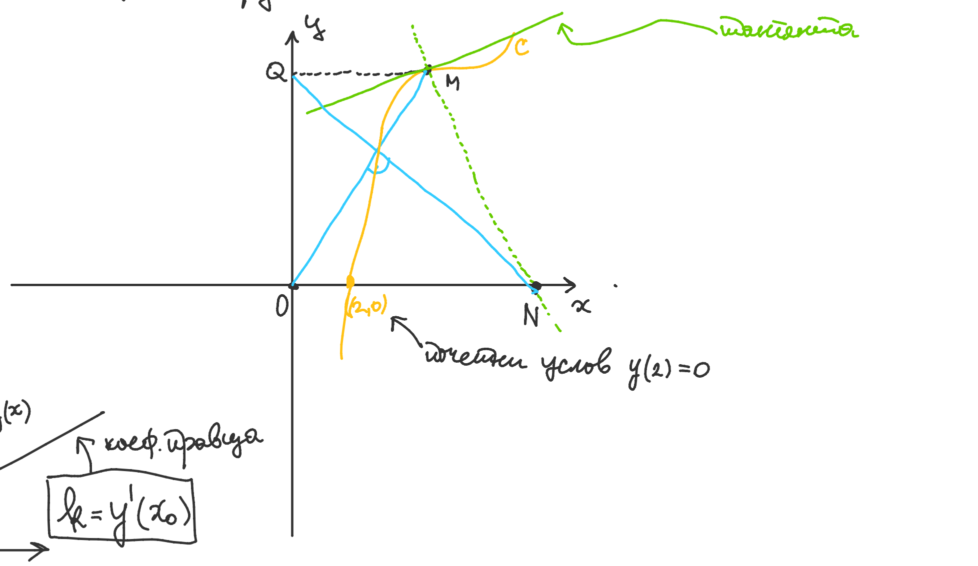
$\dfrac{dt}{du} = 2u$

$$\frac{dy}{du} \cdot \frac{\cancel{e^y}}{\cancel{2u}} = \frac{\cancel{e^y} \cdot (y^2 + u^2)}{\cancel{2}u^{3\,2}} \Rightarrow \frac{dy}{du} = \frac{y^2+u^2}{u^2} = \left(\frac{y}{u}\right)^2 + 1 \quad (\text{хом})$$

$y(u) \rightsquigarrow x(t) = e^{y(t)} = e^{y(u^2)}$

6) За сваку тачку M криве c важи следеће. Нека се M пројектује на y-осу у Q, а нормала на c у M сече x-осу у N. Нека је O коорд. почетак. Ако важи $QN \perp OM$ и крива c пролази кроз $(2,0)$, одредити c.

$c: y(x)$



Где је овде ДЈ?