Консултације септембар

$$3x^3 - e^{-3tx} + (2x + 3tx^2)x' = 0,$$

$$\frac{dx}{dt}$$

$$(3x^3 - e^{-3tx})\,dt + (2x + 3tx^2)\,dx = 0$$

$$x_1' = 4x_1^2 x_2 - x_1 E(x_1, x_2) = 0 \qquad E(x_1,x_2) = x_1^2 + 2x_2^2 - 4$$

$$x_2' = -2x_1^3 - x_2 E(x_1, x_2) = 0$$

а) $x_1 \underbrace{\left(4x_1x_2 - E(x_1,x_2)\right)} = 0$

1° $x_1 = 0 \Rightarrow x_2 E(0, x_2) = 0$

$$\boxed{1.1^\circ \ (0,0) \qquad 1.2^\circ \ (0, \pm\sqrt{2})}$$

2° $E = 4x_1x_2$

$$-2x_1^3 - x_2 \cdot 4x_1x_2 = 0$$

$$2x_1(-x_1^2 - 2x_2^2) = 0$$

2.1° $x_1 = 0$ .... исто

2.2° $x_1^2 + 2x_2^2 = 0 \rightarrow (0,0)$

б) $F_1 = 4x_1^2 x_2 - x_1^3 - 2x_1x_2^2 + 4x_1$

$F_2 = -2x_1^3 - x_1^2 x_2 - 2x_2^3 + 4x_2$

$$dF(0,0) = \begin{bmatrix} 8x_1x_2 - 3x_1^2 - 2x_2^2 + 4 & 4x_1^2 - 4x_1x_2 \\ -6x_1^2 - 2x_1x_2 & -x_1^2 - 6x_2^2 + 4 \end{bmatrix}_{(x_1,x_2)=(0,0)} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$\lambda_1 = \lambda_2 = 4 > 0$$

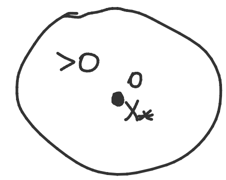
$\Rightarrow$ нестабилан

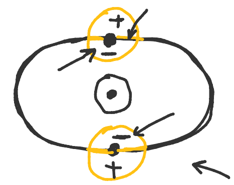
в) $E(x_1, x_2) = V(x_1, x_2)$ $\qquad$ $\underline{(0,0)}$

$(0, \pm\sqrt{2})$

$V(0,0) = E(0,0) = -4 \neq 0$ $\quad$ X

$V(0,\pm\sqrt{2}) = E(0,\pm\sqrt{2}) = 0^2 + 2(\pm\sqrt{2})^2 - 4 = 0$





$(0,-\sqrt{2}+\varepsilon)$

$(0,\sqrt{2}), \rightsquigarrow (0, \sqrt{2}-\varepsilon)$

$$E(0,\sqrt{2}-\varepsilon) = 0^2 + 2(\sqrt{2}-\varepsilon)^2 - 4 = 2\underbrace{\Big(\overbrace{(\sqrt{2}-\varepsilon)^2}^{<2} - 2\Big)}_{<0} < 0 \quad \lightning$$

$(0,\sqrt{2}-\varepsilon) \in \mathcal{U}(0,\sqrt{2})$

$$\rightarrow \underline{(x+4y)} z'_x + \underline{(-x+5y)} z'_y + 2z = x$$

$x - 2z$

$x = 2y$

$z = 5y + \frac{x}{5}$

$x(t), y(t), z(t)$

$x' = x + 4y$

$y' = -x + 5y$

$z' = x - 2z$

$X' = AX$ $\qquad$ $A = \begin{bmatrix} 1 & 4 & 0 \\ -1 & 5 & 0 \\ 1 & 0 & -2 \end{bmatrix}$

$\lambda_1 = -2, \quad \lambda_2 = \lambda_3 = 3$

$$J = \begin{bmatrix} -2 & & \\ & \boxed{3} & \boxed{?} \\ & & \boxed{3} \end{bmatrix}$$

$(A + 2E)\gamma_1 = 0$

$\vdots$

γ_1

$$P = \begin{bmatrix} \gamma_1 \downarrow & \gamma_2 \downarrow & \gamma_3 \downarrow \end{bmatrix}$$

$(A-3E)\gamma_2 = 0$

$\vdots$

$$\begin{bmatrix} -2 & 4 & 0 \\ -1 & 2 & 0 \\ 1 & 0 & -5 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

$-2a + 4b = 0$

$a - 5c = 0$

$$\gamma_2 = \begin{bmatrix} 5 \\ 2.5 \\ 1 \end{bmatrix} \sim \begin{bmatrix} 10 \\ 5 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix} \text{ 1 Жорд. блок}$$

$$\begin{bmatrix} 5c \\ 2.5c \\ c \end{bmatrix}$$

$\dim\ker(A-3E) = 1$

γ_3 уточнени?

$(A-3E)\gamma_3 = \gamma_2$

$-2a + 4b = 10$

$a - 5c = 2$

$-4 - 10c + 4b = 10$

$b = \dfrac{14+10c}{4}$

$$\begin{bmatrix} 2+5c \\ \frac{14+10c}{4} \\ c \end{bmatrix} \underset{c=1}{=} \begin{bmatrix} 7 \\ 6 \\ 1 \end{bmatrix}$$

$$J = \begin{bmatrix} \boxed{-2} & & \\ & \boxed{\begin{matrix} 3 & 1 \\ & 3 \end{matrix}} \end{bmatrix} \rightsquigarrow e^{Jt} \overset{?}{=} \begin{bmatrix} e^{-2t} & 0 & 0 \\ 0 & e^{3t} & e^{3t}t \\ 0 & 0 & e^{3t} \end{bmatrix}$$

ОР: $X(t) = e^{At} \cdot c, \ c \in \mathbb{R}^3$

$= P e^{Jt} P^{-1} \cdot c$

$$D + N$$

$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$e^{Jt} = \begin{bmatrix} e^{-2t} & \\ & \boxed{e^{\begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix} t}} \end{bmatrix} = e^{\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} t} \cdot e^{\underset{N}{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}} t} = \begin{bmatrix} e^{3t} & 0 \\ 0 & e^{3t} \end{bmatrix} \cdot (E + tN) = e^{3t} \cdot E \cdot \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

$$N^2 = 0$$

$$= \begin{bmatrix} e^{3t} & te^{3t} \\ 0 & e^{3t} \end{bmatrix}.$$

$x = 2y$

$z = \sin y + \dfrac{x}{5}$

$$\Big(\underset{\substack{x_0(s) \\ \| \\ x(0,s)}}{2s}, \ \underset{y_0(s)}{s}, \ \underset{z_0(s)}{\sin s + \tfrac{2s}{5}} \Big)$$

$(x+2y)y' = ky - \frac{2}{x}(y^4 + xy) - x$

$(k,m) \in \mathbb{Z}^2$, $z = \underline{y^m + xy} \rightsquigarrow$ мин.

$$z' = \boxed{m y^{m-1} \cdot y'} + y + \underline{x y'}$$

$$\underbrace{(x+2y)y'}_{} = ky - \frac{2}{x} z - x$$

$\parallel$

$\underline{xy'} + \boxed{2yy'}$

$m = 2$: $z' = \underbrace{2yy' + xy'}_{} + y \implies \underbrace{z' - y}_{} = \underbrace{ky}_{} - \frac{2}{x} z - x$; $k = -1$

$$\longrightarrow z' + \frac{2}{x} z = -x$$

$(2,-1)$

$e^{\alpha t}$ $s =$ вишеструкост $\alpha \pm i\beta$ као решења каракт. јне

$\alpha = 2, \beta = 0$ $(\lambda - 2)^{\boxed{2}}(\lambda - 1)$

$\alpha \pm i\beta = \underline{2}$

$\lambda_1, \lambda_2, \lambda_3$

$$\begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \lambda_3 & \\ & & & \ddots \end{bmatrix}$$

$\alpha_1 \pm i\beta_1, \ldots$

$$\begin{bmatrix} \alpha_1 & -\beta_1 \\ \beta_1 & \alpha_1 \end{bmatrix}$$

$2, -3, 1 \pm i, 7, 2 \pm 3i$

$$J = \begin{bmatrix} \boxed{2} & & & & & & \\ & \boxed{-3} & & & & & \\ & & 1 & -1 & & & \\ & & 1 & 1 & & & \\ & & & & \boxed{7} & & \\ & & & & & 2 & -3 \\ & & & & & 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & \boxed{1?} & \\ & 3 & \boxed{1?} \\ & & 3 \end{bmatrix}$$

$(A-3E)^k = 0$

$\vdots$

$\dim\ker(A-3E) \rightsquigarrow 1 \to 1$ Ж.б.

$\rightsquigarrow 2 \to 2$ Ж.б.

$\rightsquigarrow 3 \to 3$ Ж.б.

$$\begin{bmatrix} 3 & 1 & \\ & 3 & 1 \\ & & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & \\ & 3 & \\ & & \boxed{3} \end{bmatrix} \qquad \boxed{3}^{0} \begin{bmatrix} 3 & 1 \\ & 3 \end{bmatrix}$$

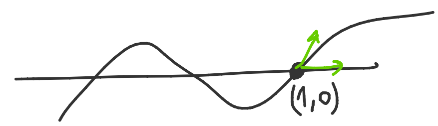
$$\begin{bmatrix} \boxed{3} & & \\ & \boxed{3} & \\ & & \boxed{3} \end{bmatrix}$$

$$y'' + \underline{a(x)}y' + \underline{b(x)}y = \underline{0}$$

$(-1-\varepsilon, -1+\varepsilon)$

$$y(x) = (x-1)^2 P(x)$$

Пикарова Т !



1. реш: $y \equiv 0$, $\underline{y(1) = y'(1) = 0}$

2. реш: $y(x) = (x-1)^2 P(x)$

$\underline{y(1) = y'(1) = 0}$

$y(1) = 0$

$y'(x) = \underline{(x-1)^2} P'(x) + \underline{2(x-1)} \cdot P(x)$

$y'(1) = 0$

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3. Нека су $a(t)$ и $b(t)$ непрекидне функције на $\mathbb{R}$ и нека су $x_1(t)$ и $x_2(t)$ два решења једначине

$$x''(t) + a(t)x'(t) + b(t)x(t) = 0$$

која задовољавају почетне услове $x_1(0) = 0,\ x_1'(0) = 2,\ x_2(0) = 0,\ x_2'(0) = 7$. Доказати да су функциј[е x_1 и] x_2 пропорцијалне, односно да постоји $c \in \mathbb{R}$ такво да је $x_1(t) = cx_2(t)$.

Вронскијан

$$W(x_1,x_2)(t) = \begin{vmatrix} x_1(t) & x_2(t) \\ x_1'(t) & x_2'(t) \end{vmatrix} = x_1 x_2' - x_2 x_1'$$

$$W(x_1,x_2)(0) = \begin{vmatrix} 0 & 0 \\ 2 & 7 \end{vmatrix} = 0 \Rightarrow x_1, x_2 \text{ су лин. зав.}$$