

2. Решити парцијалну диференцијалну једначину  $\underbrace{z(x+y)}_{\frac{\partial u}{\partial x}} + z(y-x) \frac{\partial u}{\partial y} - (x^2+y^2) \frac{\partial u}{\partial z} = 0$ .

$$\frac{dx}{z(x+y)} = \frac{dy}{z(y-x)} =$$

$$\frac{x dx + y dy}{z(x^2 + y^2) + z(y^2 - x^2)} = \frac{-dz}{x^2 + y^2}$$

$$\rightarrow \frac{dx}{dy} = \frac{x+y}{y-x}$$

$$x' = \frac{\frac{x}{y} + 1}{1 - \frac{x}{y}}$$

$$\underbrace{x dx + y dy} = - \underbrace{z dz}$$

3. Решити Кошијев проблем  $\underbrace{y(x+y)}_{\frac{\partial u}{\partial x}} - \underbrace{x(y+x)}_{\frac{\partial u}{\partial y}} + \underbrace{(x-y)(2x+2y+z)}_{\frac{\partial u}{\partial z}} = 0$ ,  $u(1, y, z) = 2y + z + yz$ .

$$\frac{dx}{y} = - \frac{dy}{z} \checkmark$$

$$z(x-y) + 2(x^2 - y^2)$$

$$\frac{dx + dy}{y^2 - x^2} = \frac{dz}{(x-y)(2x+2y+z)}$$

$$\frac{d(x+y)}{x+y} = \frac{dz}{2(x+y)+z}$$

"мера"

$$x+y=t$$

$$\frac{dt}{t} = \frac{dz}{2t+z}$$

$$\frac{dz}{dt} = \frac{2t+z}{t} = 2 + \frac{z}{t} \dots$$

4. Решити парцијалну диференцијалну једначину  $\underbrace{(x^2 y^2 + x^3 + x)}_{\frac{\partial u}{\partial x}} + \underbrace{(1 - x^2)}_{\frac{\partial u}{\partial y}} + \underbrace{xz(xy^3 + x^2 y + y + 1 - x^2)}_{\frac{\partial u}{\partial z}} = 0$ .

$$\frac{y dx}{y f_1}$$

$$\frac{y dx + x dy}{\dots} = \dots \checkmark$$

$$\frac{dx}{x^2 y^2 + x^3 + x} = \frac{dy}{1 - x^2}$$

5. Решити парцијалну диференцијалну једначину  $(2x^2z^2 + x)\frac{\partial u}{\partial x} - (4xyz^2 - y)\frac{\partial u}{\partial y} - (4xz^3 - z)\frac{\partial u}{\partial z} = 0$  и одредити решење  $u = u(x, y, z)$  тако да је  $u = yz^2$  при услову  $x = z$ .

$$x(2xz^2+1), y(-4xz^2+1), z(-4xz^2+1)$$

$$\int (2x^2z^2+x)dz = \int (-4xz^3+z)dx$$

$$F(x,z) = xz + x^2z^3 \cdot \frac{z}{3}$$

$$\mu(x,z) = \mu(w(x,z))$$

$$\frac{zdx}{2x^2z^3+xz} = \frac{x dz}{-4x^2z^3+zx}$$

$$\frac{\alpha \frac{dx}{x} - \beta \frac{dz}{z}}{\alpha(2xz^2+1) + \beta(-4xz^2-1)} = \frac{\frac{dy}{y}}{-4xz^2-1}$$

$$xz^2: 2\alpha + 4\beta = -4$$

$$1: \alpha - \beta = -1$$

$$3(x-6)^2y'' + 25(x-6)y' - 16y = x-16, \quad y(4) + 3y'(4) = 1, \quad y'(5) + 8 = 8y(5).$$

$$y(4) + 3y'(4) = 1$$

$$8y(5) - y'(5) = 8$$

$$|y=1|$$

$$z = y-1, y = z+1$$

$$3(x-6)^2z'' + 25(x-6)z' - 16z = \cancel{x}$$

$$\begin{aligned} z(4) + 3z'(4) &= 0 \\ 8z(5) - z'(5) &= 0 \end{aligned}$$

$$3(x-6)^2z'' + 25(x-6)z' - 16z = 0$$

⋮ (општење)

$$C_1 \cdot (x-6)^{2/3} + C_2 \cdot (x-6)^{-8}$$

$$\begin{aligned} z_1(x) &, z_1(4) = -3, z_1'(4) = 1 \\ z_2(x) &, z_2(5) = 1, z_2'(5) = 8 \end{aligned}$$

$$G(x,t) = \dots$$

$$\int G \cdot x \, dt = \dots$$

$$x_1' = ax_1 + 2x_2$$

$$x_2' = -3x_2$$

$$a \in \mathbb{R}$$

а)

б)

$$A = \begin{bmatrix} a & 2 \\ 0 & -3 \end{bmatrix}$$

$$\lambda_1 = a$$

$$\lambda_2 = -3$$

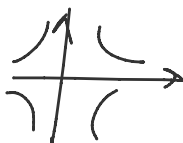
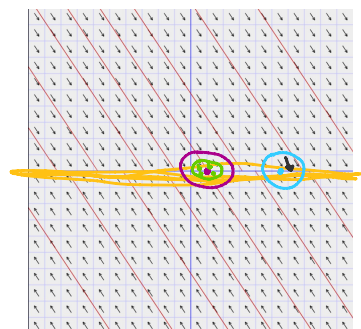
1°  $a < 0, a \neq -3$

асим.  
центр

2°  $a = -3$  → гетер. узор

3°  $a = 0 \rightarrow (1, 0), t \in \mathbb{R}$

нест. — 4°  $a > 0$



асим., не асимпт.

$$xy'' + 2y' + xy = 1$$

$$\psi_1(x) = \frac{\sin x}{x}$$

$$\frac{dT_1}{dt} = -a(T_1(t) - T_\infty),$$

$$\frac{dT_1}{T_1 - T_\infty} = -a \, dt \quad / \int$$

$$\ln|T_1 - T_\infty| = -at + c$$

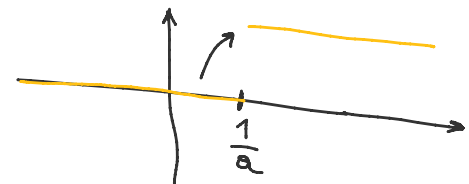
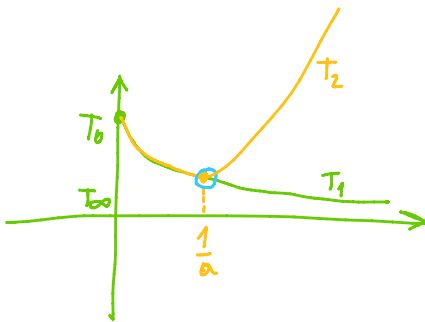
$$T_1 = T_\infty + c \cdot e^{-at}$$

$$T_1(0) = T_0$$

$$T_0 = T_\infty + c \cdot 1 \Rightarrow c = T_0 - T_\infty$$

$$T_1 = T_\infty + (T_0 - T_\infty) \cdot e^{-at}$$

$$\frac{dT_2}{dt} = -a(T_2(t) - T_\infty) + tH\left(t - \frac{1}{a}\right),$$



$$\frac{dT_2}{dt} = -a(T_2 - T_\infty) + \underline{t} \quad - \text{un. gf.}$$

⋮

$$T_2(t), t > \frac{1}{a}$$

$$C_2 = ?$$

непрерывна!

$$T_2\left(\frac{1}{a}-\right) = T_2\left(\frac{1}{a}+\right)$$

$$(x+4y)z'_x + (-x+5y)z'_y + 2z = x$$

$$x(t), y(t), z(t)$$

$$\left. \begin{aligned} x' &= x+4y \\ y' &= -x+5y \\ z' &= x-2z \end{aligned} \right\}$$

$$A = \begin{bmatrix} 1 & 4 & 0 \\ -1 & 5 & 0 \\ 1 & 0 & -2 \end{bmatrix} \dots$$

$$x = 2y$$

$$z = 5y + \frac{x}{5}$$