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$10x+1 \le p \le 10x+10$, $5 \le x \le 9$ $\longrightarrow$ оцена $x+1$

план курса:
- увод
- функције
- низови
- редови
- интеграли

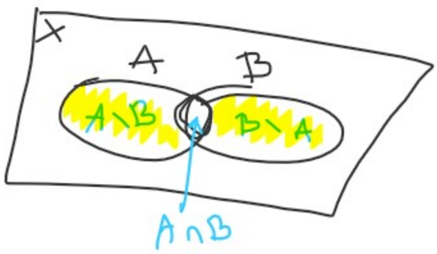
Литература: *предавања и вежбе
- скрипта Анализа 1, Дарко Милинковић
- Лашко 1,2
- Анализа 1, Стојан Раденовић
- Анализа 1, Ђорђе Кртинић

*рокови

A, B скупови, $A, B \subseteq X$

- $A \subseteq B \iff (x \in A \Rightarrow x \in B)$
- $A = B \iff A \subseteq B \wedge B \subseteq A$
- $A \cap B = \{x \in X \mid x \in A \wedge x \in B\}$
- $A \cup B = \{x \in X \mid x \in A \vee x \in B\}$
- $A \setminus B = \{x \in X \mid x \in A \wedge x \notin B\}$
- $A \triangle B = (A \setminus B) \cup (B \setminus A)$

  симетрична разлика



$A^{c} = \{x \in X \mid x \notin A\}$

(1) $A = \{(x,y) \in \mathbb{R}^2 \mid |x|+|y| < 1\}$ $\longrightarrow$ $\mathbb{R}$ - скуп реалних бројева

$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(x,y) \mid x \in \mathbb{R} \wedge y \in \mathbb{R}\}$

$B = \{(x,y) \in \mathbb{R}^2 \mid \sqrt{x^2+y^2} < 1\}$

$C = \{(x,y) \in \mathbb{R}^2 \mid |x+y| < \sqrt{2}\}$

Доказати $A \subseteq B \subseteq C$.

1) $A \subseteq B$ 2) $B \subseteq C$

1) $t \in A \Rightarrow t \in B$

$(x,y) \in A \overset{?}{\Rightarrow} (x,y) \in B$

2) $\sqrt{x^2+y^2} < 1 \overset{?}{\Rightarrow} |x+y| < \sqrt{2}$

$\iff$ $\iff$

$(x,y)\in A \overset{?}{\Rightarrow} (x,y)\in B$

$|x|+|y|<1 \;/^2 \qquad \sqrt{x^2+y^2}<1$

$\Downarrow$

$x^2+y^2 \le x^2+\boxed{2|x|\cdot|y|}+y^2<1$

$x^2+y^2<1$

$\Leftrightarrow$

$x^2+y^2<1$

$$\boxed{x^2+y^2\geq 2xy}$$

$x^2-2xy+y^2\geq 0$

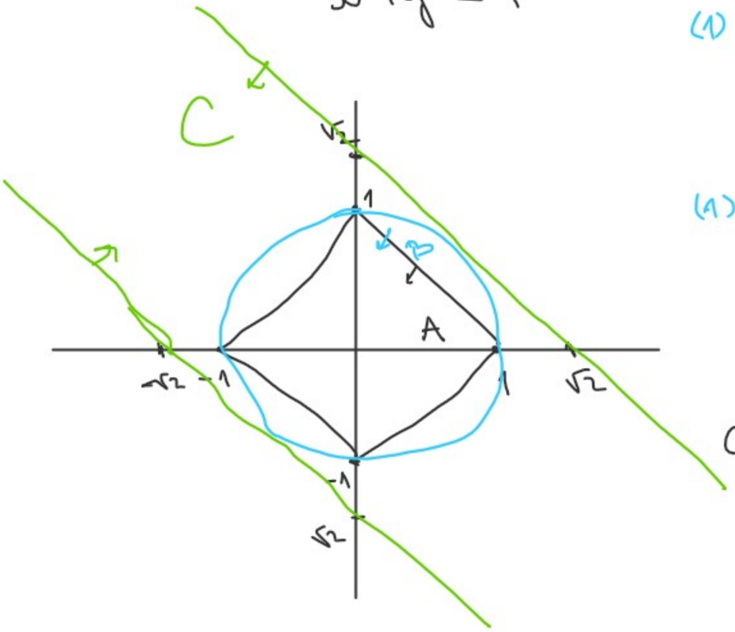
$(x-y)^2\geq 0$

$\Leftrightarrow$

$(x+y)^2<2$

$\Leftrightarrow$

$x^2+y^2+\boxed{2xy}<2$



(1) $x^2+y^2<1$

$\left.\begin{array}{l} x^2+y^2<1 \\ x^2+y^2\geq 2xy \end{array}\right\} \Rightarrow 2xy\le x^2+y^2<1$

$\overset{(2)}{\Rightarrow} 2xy<1$

(1)+(2): $x^2+y^2+2xy<2$

$(x+y)^2<2$

$|x+y|<\sqrt{2} \Rightarrow (x,y)\in C$

C: $|x+y|<\sqrt{2}$

$-\sqrt{2}<x+y<\sqrt{2}$

Нотација:

- $\bigcup_{k=1}^{n} A_k = A_1\cup A_2\cup\ldots\cup A_n=\{x\in X \mid \underline{x\in A_1 \vee \ldots \vee x\in A_n}\}$

  $(\exists k\in\{1,\ldots,n\})\; x\in A_k$

- $\bigcup_{k=1}^{\infty} A_k = A_1\cup A_2\cup\ldots = \{x\in X \mid (\exists k\in\mathbb{N})\; x\in A_k\}$

$\bigcup_{i\in I} A_i = \{x\in X \mid \underline{(\exists j\in I)\; x\in A_j}\}$

пр: $A_x=\{x\}$

$\bigcup_{x\in\mathbb{R}} A_x=\mathbb{R}$

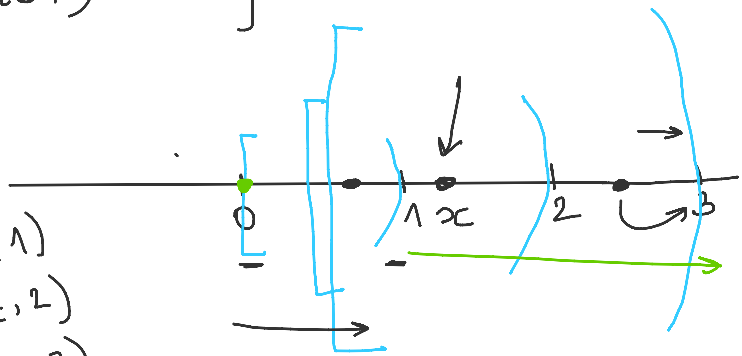
- $I=\{1,\ldots,n\}$
- $I=\{1,2,\ldots\}=\mathbb{N}$

$\bigcap_{k=1}^{n} A_k = A_1\cap\ldots\cap A_n=\{x\in X \mid (\forall k\in\{1,\ldots,n\})\; x\in A_k\}$

$\bigcap_{k=1}^{\infty} A_k = \bigcap_{k\in\mathbb{N}} A_k = \{x\in X \mid (\forall k\in\mathbb{N})\; x\in A_k\}$

$$\bigcap_{i\in I} A_i = \{x\in X \mid (\forall i\in I)\, x\in A_i\}$$

② $A_n = [1-\frac{1}{n}, n)$, $n\in\mathbb{N}$. Нађи $\bigcup_{n\in\mathbb{N}} A_n$.



$A_1 = [0,1)$

$A_2 = [\frac{1}{2}, 2)$

$A_3 = [\frac{2}{3}, 3)$ ...

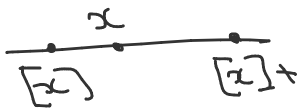
$1-\frac{1}{n} \to 1$, $n\to\infty$ Имамо: (!) $\bigcup_{n\in\mathbb{N}} A_n = [0,+\infty)$. $1-\frac{1}{n} \geq 0$

$\subseteq$: $x\in\bigcup_{n\in\mathbb{N}} A_n \Rightarrow (\exists n\in\mathbb{N})\, x\in A_n \Rightarrow (\exists n\in\mathbb{N})\, x\in[1-\frac{1}{n}, n) \subseteq [0,+\infty)$

$\Rightarrow x\in[0,+\infty)$

$\supseteq$: $x\in[0,+\infty) \overset{?}{\Rightarrow} (\exists n\in\mathbb{N})\, x\in[1-\frac{1}{n}, n)$

1° $x\geq 1$: $x\in A_{[x]+1}$



$[x]$ – највећи цео број $\leq x$.

2° $0\leq x<1$. $x\in A_1$

Домаћи: $\bigcap_{n\in\mathbb{N}} A_n = ?$ $(\emptyset)$

Деф. A, B. $\mathcal{P}(A) = \{X \mid X\subseteq A\}$. $\mathcal{P}(\{1,2\}) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

$A\times B = \{(x,y) \mid x\in A \wedge y\in B\}$

③ Доказати:

а) $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

б) $\left(\bigcup_{i\in I} A_i\right)^C = \bigcap_{i\in I} A_i^C$ – Де Морганов закон

$x\notin A \Leftrightarrow \neg(x\in A)$

$\left(\bigcap A_i\right)^C = \bigcup A_i^C$

в) $(A\times B)\cap(C\times D) = (A\cap C)\times(B\cap D)$

г) $\mathcal{P}(A\cap B) = \mathcal{P}(A)\cap\mathcal{P}(B)$

в) $(A\times B)\cap(C\times D) = (A\cap C) \ldots$

г) $\mathcal{P}(A\cap B) = \mathcal{P}(A)\cap\mathcal{P}(B)$

$$\Big(\bigcap_{i\in I} A_i\Big)^c = \bigcup_{i\in I} A_i^c$$

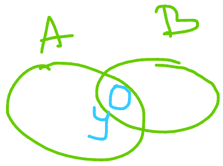
б) $x\in\Big(\bigcup_{i\in I}A_i\Big)^c \iff x\notin\bigcup_{i\in I}A_i \iff \neg\Big(x\in\bigcup_{i\in I}A_i\Big)$

$\iff \neg\big((\exists i\in I)\, x\in A_i\big)$

$\iff (\forall i\in I)\, x\notin A_i$

$\iff (\forall i\in I)\, x\in A_i^c$

$\iff x\in\bigcap_{i\in I}A_i^c$



г) $Y\in\mathcal{P}(A\cap B) \iff Y\subseteq A\cap B \iff Y\subseteq A \wedge Y\subseteq B$

$\iff Y\in\mathcal{P}(A) \wedge Y\in\mathcal{P}(B)$

$\iff Y\in\mathcal{P}(A)\cap\mathcal{P}(B)$